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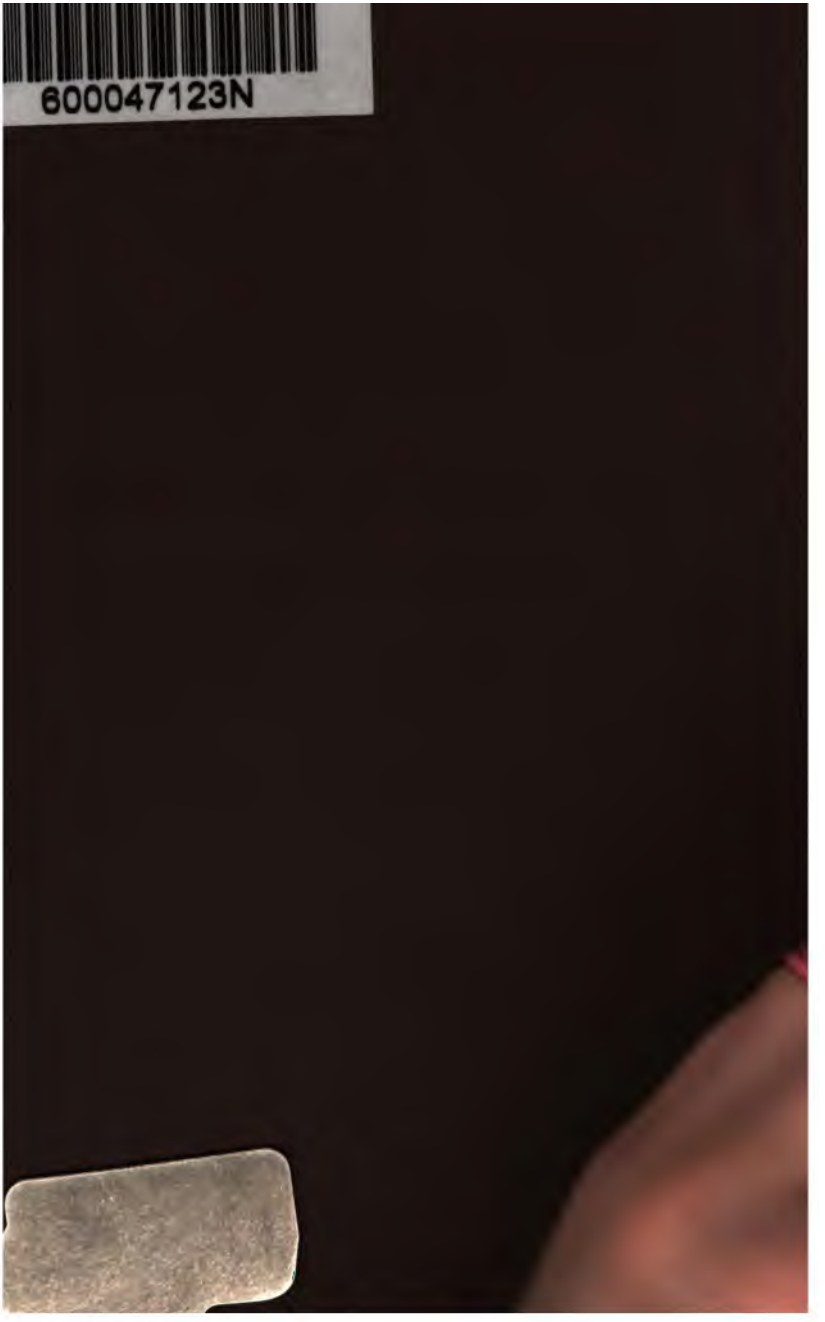
TEXT-BOOK
ON
ICAL SOLID OR DESCRIPTIVE
EOMETRY

D. A. LOW.

PART I.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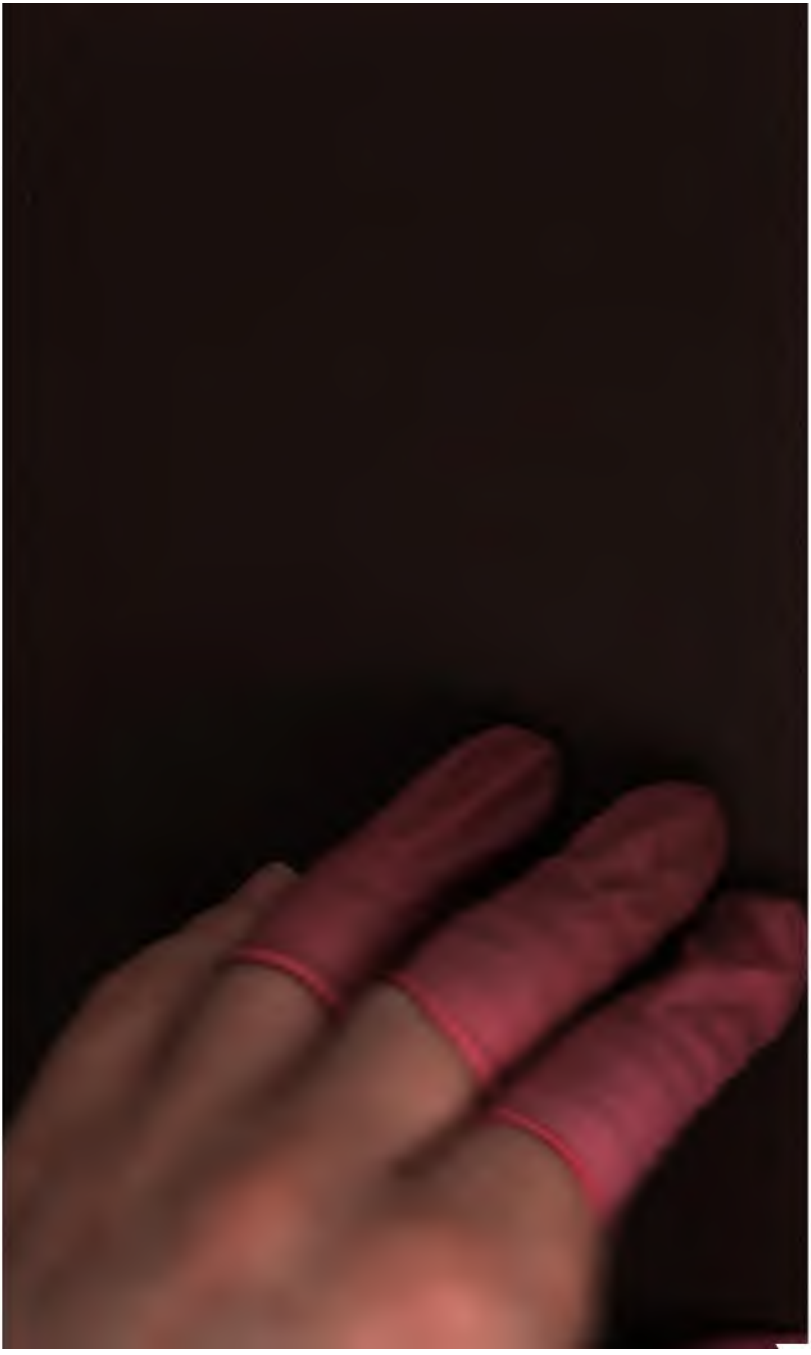

PRACTICAL SOLID OR DESCRIPTIVE

GEOMETRY

The following are the Contents of Part II. (price 3s.)

CHAPTER X.
Additional Problems on the Straight Line and Plane.

CHAPTER XI.
Projection of Solids.

CHAPTER XII.
Isometric Projection.

CHAPTER XIII.
Horizontal Projection.

CHAPTER XIV.
Curved Surfaces and Tangent Planes.

CHAPTER XV.
Developments and the Projection of Screw Threads.

CHAPTER XVI.
Intersection of Surfaces.

CHAPTER XVII.
Projection of Shadows.

CHAPTER XVIII.
Miscellaneous Problems.

CHAPTER XIX.
Theory of Perspective.

APPENDIX.

TEXT-BOOK

ON

PRACTICAL SOLID OR DESCRIPTIVE

GEOMETRY

BY

DAVID ALLAN LOW

(WHITWORTH SCHOLAR)

LECTURER ON ENGINEERING IN ALLAN GLEN'S INSTITUTION, GLASGOW

IN TWO PARTS—PART I.

LONDON

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1884

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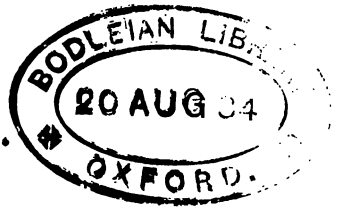

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PREFACE.

THE AUTHOR, in writing this text-book, has endeavoured to meet the wants of both elementary and advanced students, and he believes that it will be found to contain all the descriptive geometry which is usually required by engineering and architectural draughtsmen. But while making the book comprehensive, and illustrating it fully, it has not been made of an inconvenient size for use in large classes.

The treatment of the subject in this work is slightly different from that in any existing books. The problems are stated in a more comprehensive way, and are made to include more cases than is usual with other writers. After the statement of the problem follows the general solution, which is usually given without reference to any particular example. Next comes the application of the problem to one or more examples. In many cases the student may not fully understand the general solution of a problem until he has worked out the examples which illustrate it. The advantage of this mode of treatment is, that it is more systematic, and enables the student to get a more intelligent and comprehensive grasp of the subject. After working the examples and mastering the general solution of a problem, the student is better able to cope with any fresh examples which may come before him, than if he had learned the subject from examples only.

The elementary portion of the subject is treated of in Part I., and the more advanced portion in Part II.

A great want which the author has found in existing works on descriptive geometry is that of a sufficient number of good exercises properly graduated; he has, therefore, been at considerable trouble to collect and devise a large number of exercises, and he believes that in no other work of the kind will there be found such a good collection. In this matter he would record his indebtedness to the examination papers published by the Science and Art Department, which has done so much to promote the teaching of this and other science subjects throughout the country.

In conclusion, the author would like to impress upon the student the necessity of working out all the examples and exercises on paper with the drawing instruments, neatly and of full size. It is not enough for the student to know how a problem is to be solved, he must actually work it out; as very often, from the peculiar position of the points, lines, or planes, the result is quite different from what he would have expected.

D. A. L.

GLASGOW: *November* 1883.

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PRACTICAL SOLID

OR

DESCRIPTIVE GEOMETRY.

PART I.

CHAPTER I.

INTRODUCTION.

Practical Solid or Descriptive Geometry is that branch of geometry which treats (1) of the representation of figures having three dimensions—length, breadth, and thickness—upon a plane surface, which has only two dimensions, namely length and breadth; and (2) of methods for determining from this representation the exact form of the figure represented.

Projection. The problems of Descriptive Geometry are best solved by means of the method of projections which we shall now consider.

When an object is seen by the eye of an individual, rays of light come from all the visible points of that object, and converge towards a point within the eye. Now suppose that a flat piece of glass is placed between the object and the eye of the spectator, and that each ray of light, in passing through the glass from the object to the eye leaves a mark on its surface of the same colour and tint as the part of the object from which the ray came, a picture would be produced on the surface of the glass; and if the object were now removed,

while the picture and the eye remain stationary, this picture would convey to the mind of the spectator the same knowledge of the object as was conveyed by the presence of the object itself. Again, if instead of the rays of light from all the visible points of the object leaving an impression on the glass, only those which came from the edges of the object were to do so, we would now have an outline on the surface of the glass which, although it did not convey to the mind the same impression as the presence of the object itself, might still give a good idea of its form.

The foregoing remarks are illustrated by fig. 1, where A B represents an object viewed by an eye at E; C D is a plane interposed between E and A B; the dotted lines represent a few of the rays of light passing from the object to the eye, and A′ B′ is the outline obtained from the intersections of the rays of light with the plane.

The figure A′ B′ is called a *projection* of the object A B upon the plane C D.

Projectors. The rays of light or imaginary lines passing from the different points of the object to the corresponding points in the projection are called *projectors.*

Perspective Projection. When the projectors converge to a point the projection is a *radial, conical,* or *perspective* projection.

Parallel Projection. When the point to which the projectors converge is at an infinite distance from the object the projectors become parallel, and the projection is a *parallel* projection.

Orthographic Projection. If besides being parallel the projectors are also perpendicular to the plane of projection, the projection is a *perpendicular,* an *orthogonal,* or *orthographic* projection.

The greater part of this treatise will be taken up with the study of the latter kind of projection, and after this, when the word 'projection' is used without any qualification, orthographic projection is to be understood.

The *Projection of a Point* upon a plane is the foot of the perpendicular let fall from the point on the plane.

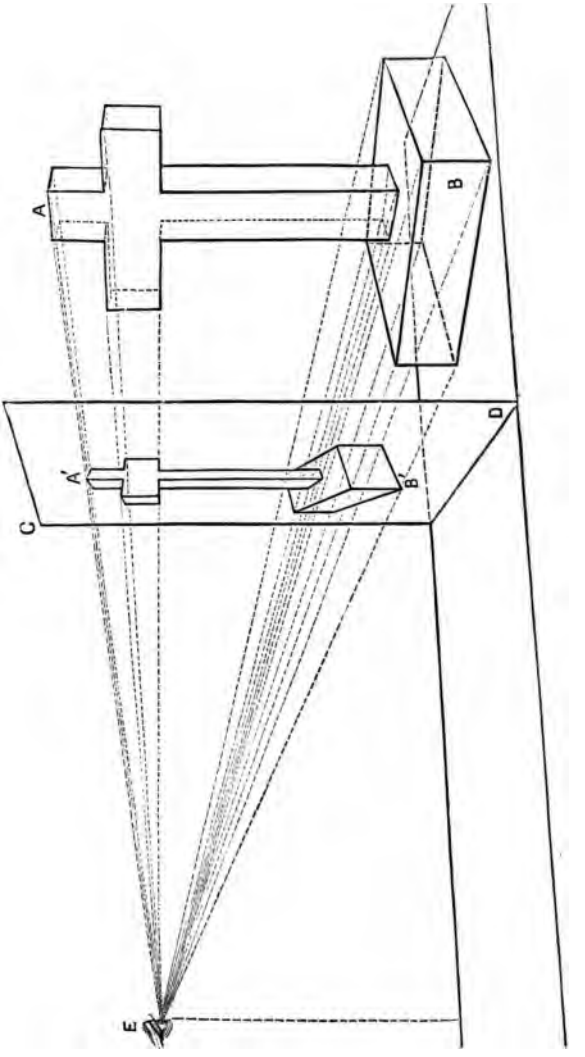


FIG 1.

The *Projection of a Line* upon a plane is the line containing the projections of all the points of that line.

The *Projecting Surface* of a line is the surface which contains the projectors of all the points of that line.

When the projecting surface is a plane it is called the *projecting plane.*

One projection alone of a figure is not sufficient for determining its exact form. For instance, let the triangle $a\,b\,c$ be the projection on this sheet of paper of a triangle, which is situated, say, somewhere above it. It is clear that the exact form of the triangle of which $a\,b\,c$ is a projection will depend upon the relative distances of its angular points from the paper; but the projection $a\,b\,c$ tells us nothing about these distances. If, however, we take another projection of the triangle, say, on a sheet of paper at right angles to the former one, we

FIG. 2.

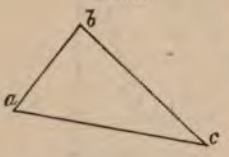
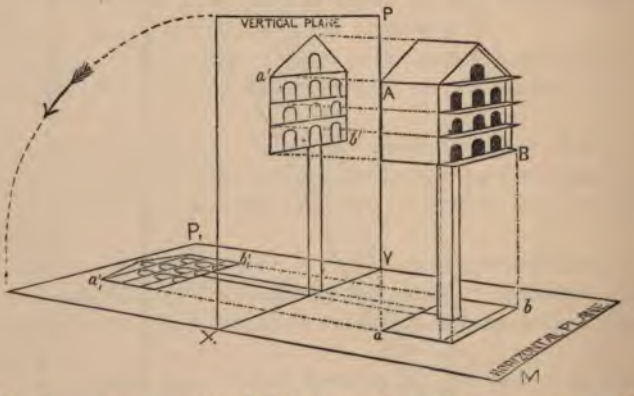


FIG. 3.



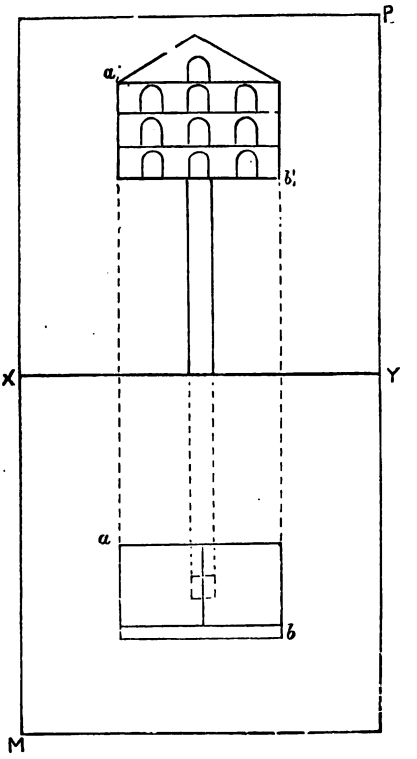
will show as we proceed that from these two projections the true form of the figure whatever it may be can be determined.

We have already stated that the problems of Descriptive Geometry are solved by means of the method of projections;

but we have also just stated that two projections, one on each of two planes at right angles to one another, were required. We must now show how these projections can be drawn on one sheet of paper.

Referring to fig. 3, *a b* represents the projection of the dovecot A B upon the horizontal plane Y M, *a' b'* represents another projection of the cot on the vertical plane P X. These two projections *a b*, *a' b'* taken together completely determine the form of the dovecot A B. In order to bring these two projections into the same plane, let the plane P X rotate about the line X Y until it comes into the horizontal position P_1X; the two projections will now be in the same plane P_1M, and may be drawn as in fig. 4.

FIG. 4.



Co-ordinate Planes. The horizontal and vertical planes upon which figures are projected in orthographic projection are called *co-ordinate planes.* These planes are supposed to be of indefinite extent.

Ground Line. The line of intersection of the co-ordinate planes is called the *axis* or *ground line*, and is usually denoted by the letters X Y.

Plan and *Elevation.* The projection on the horizontal

plane is called a *plan*, and the projection on the vertical plane is called an *elevation*.

Notation. A capital letter denotes a point, a small letter its plan, and a small letter with a dash its elevation; thus p denotes the plan, and p' the elevation of the point P.

If A B is a line in space, its plan would be denoted by $a\,b$, and its elevation by $a'\,b'$.

In speaking of a point P in space, we may call it simply the point P, or the point $p\,p'$. In like manner in speaking of a line A B, we may call it the line $a\,b$, $a'\,b'$.

The abbreviations V.P. and H.P. stand for vertical plane and horizontal plane respectively.

In fig. 4 it will be observed that the plan is below and the elevation above X Y; this arises from the fact that the object is above the horizontal plane and in front of the vertical plane, as shown in fig. 3, but this need not be the case; the figure to be projected may occupy any position whatever with reference to the co-ordinate planes.

CHAPTER II.

PROJECTION OF POINTS AND LINES.

PROBLEM 1.

Given the position of a point with reference to the co-ordinate planes; to determine its projections.

First, to find the plan of the point.

If the point is *in front* of the vertical plane its plan is *below* X Y, and if the point is *behind* the vertical plane its plan is *above* X Y.

The distance of the plan from X Y is the same as the distance of the point from the vertical plane.

Second, to find the elevation of the point.

The plan and elevation of a point are in the same straight line perpendicular to X Y.

The elevation of the point is *above* or *below* X Y according as the point is *above* or *below* the horizontal plane.

The distance of the elevation from X Y is the same as the distance of the point from the horizontal plane.

The student must verify and carefully study the above statements by reference to fig. 5, which is a perspective view

FIG. 5.

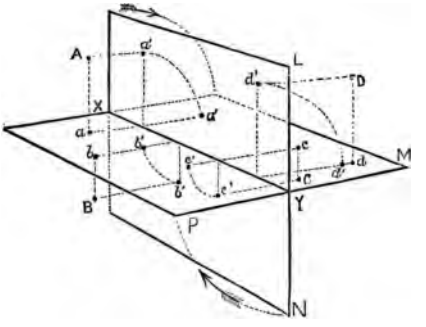
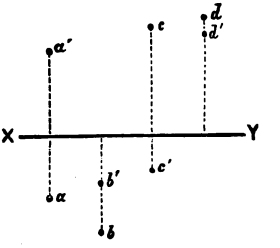


FIG. 6.



of the co-ordinate planes in their natural position, and of four points—A B C D, showing how the projections of the latter are obtained. Fig. 6 is the orthographic projection of the same four points on the plane of the paper.

The angles P Y L, L Y M, M Y N, and N Y P (fig. 5) are called the *first*, *second*, *third*, and *fourth dihedral angles* respectively.

EXAMPLE. *To find the projections of the following points:—*

A 2″ *in front of the* V.P. *and* $1\frac{1}{2}$″ *above the* H.P.
B 1″ ,, ,, ,, $1\frac{1}{2}$″ *below* ,,
C $1\frac{3}{4}$″ *behind* ,, ,, $1\frac{3}{4}$″ *above* ,,
D $\frac{1}{2}$″ ,, ,, ,, $2\frac{1}{4}$″ *below* ,,
E *in* ,, ,, 2″ ,, ,,

The plan of A will be 2″ below X Y, and its elevation $1\frac{1}{2}$″ above X Y.

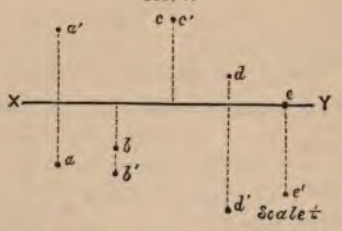
,, B ,, 1″ ,, ,, ,, $1\frac{1}{2}$″ below X Y.

The plan of C will be $1\frac{3}{4}''$ above X Y, and its elevation $1\frac{3}{4}''$ above X Y.

" D " $\frac{1}{2}''$ " " " $2\frac{1}{4}''$ below X Y.

" E " in " " " $2''$ below X Y.

FIG. 7.



The projections of the points will therefore be as shown in fig. 7.

PROBLEM 2.

To determine, from its projections, the position of a point with reference to the co-ordinate planes.

This is the converse of Problem 1.

First, to find the position of the point with reference to the vertical plane.

If the plan is *below* X Y the point is *in front* of the vertical plane, and if the plan is *above* X Y the point is *behind* the vertical plane.

The distance of the point from the vertical plane is the same as the distance of its plan from X Y.

Second, to find the position of the point with reference to the horizontal plane.

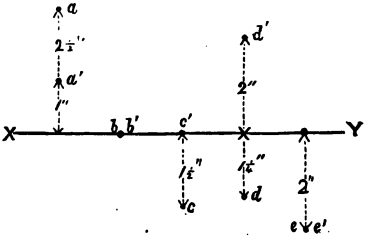
The point is *above* or *below* the horizontal plane according as the elevation is *above* or *below* X Y.

The distance of the point from the horizontal plane is the same as the distance of its elevation from X Y.

EXAMPLE. *To determine the positions of the points whose projections are given in fig.* 8:—

A is $2\frac{1}{2}''$ behind the V.P. and $1''$ above the H.P., and ∴ in the second dihedral angle.

FIG. 8.



B is in the V.P. and in the H.P., and ∴ in the ground line.

C is $1\frac{1}{2}''$ in front of the V.P. and in the H.P., and ∴ between the first and fourth dihedral angles.

D is $1\frac{1}{4}''$ in front of the V.P. and 2′ above the H.P., and ∴ in the first dihedral angle.

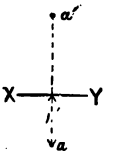
E is $2''$ in front of the V.P. and $2''$ below the H.P., and ∴ in the fourth dihedral angle.

PROBLEM 3.

Given the distance of a point from one of the co-ordinate planes, and its projection on that plane; to determine its other projection.

The other projection will lie on the line perpendicular to X Y and passing through the given projection, and its distance above or below X Y will be determined from the given distance, as in Problem 1.

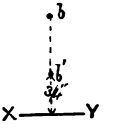
FIG. 9.



EXAMPLE 1. *a′ is the elevation of a point which is* $1''$ *in front of the vertical plane; to find its plan.*

The plan will lie on the line through *a′* perpendicular to X Y, and, since the point is $1''$ in front of the vertical plane, the plan will be $1''$ below X Y.

FIG. 10.



EXAMPLE 2. *b is the plan of a point which is* $\frac{3}{4}''$ *above the horizontal plane; to determine the elevation.*

The elevation will be $\frac{3}{4}''$ above X Y in the line through *b*, at right angles to X Y.

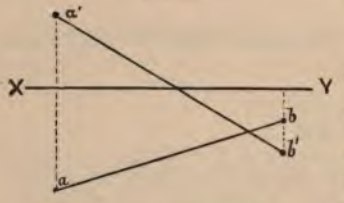
PROBLEM 4.

Given the projections of two points in a straight line on one of the co-ordinate planes, and the distances of these points from that plane; to draw the projections of the line.

By Problem 3 the other projections of the points can be determined; then the line joining the plans of the points will be the required plan of the line, and that joining the elevations of the points will be the required elevation.

EXAMPLE 1. *a, b are the plans of two points in a straight line; A is* $1\frac{1}{2}''$ *above, and B is* $1\frac{1}{4}''$ *below, the horizontal plane; to determine the plan and elevation of the line A B.*

FIG. 11.

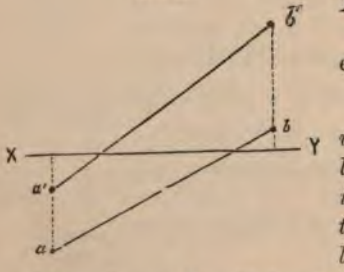


Join $a\,b$. $a\,b$ is the plan of the line.

Since A is $1\frac{1}{2}''$ above the horizontal plane, its elevation a' will be $1\frac{1}{2}''$ above X Y, and in a line through a at right angles to X Y.

The elevation b' of B will be $1\frac{1}{4}''$ below X Y in the line through b, at right angles to X Y.

FIG. 12.



The line joining $a'\,b'$ is the elevation of the line.

EXAMPLE 2. *a', b' are the elevations of two points in a straight line; A is* $2''$ *in front of, and B is* $\frac{1}{2}''$ *behind, the vertical plane; to draw the projections of the line A B.*

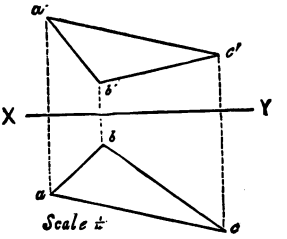
The plan of A will be $2''$ below X Y in a line through a' perpendicular to X Y.

The plan of B will be $\frac{1}{2}''$ above X Y in a line through b', at right angles to X Y.

Thus, having determined the plans of two points in the line, the line joining them will be the required plan, and the line joining the given elevations, a' and b', will be the required elevation.

EXAMPLE 3. *a b c is the plan of a triangle; the heights of the points A B C above the horizontal plane are* 2″, $\frac{1}{2}$″, *and* 1″ *respectively. To draw the elevation of the triangle.*

FIG. 13.



The elevations, a' b' c', of A B and C can be determined as in Problem 3. Joining these, the elevation of the triangle is determined.

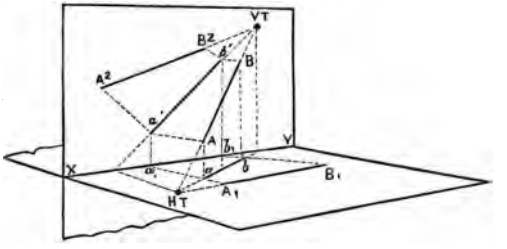
The inclination of a line to a plane is the angle between the line and its projection on that plane.

The traces of a line are the points in which the line, or line produced, meets the co-ordinate planes.

PROBLEM 5.

From the projections of a line to determine (*a*) *its length,* (*b*) *its inclinations to the co-ordinate planes,* (*c*) *its traces.*

FIG. 14.

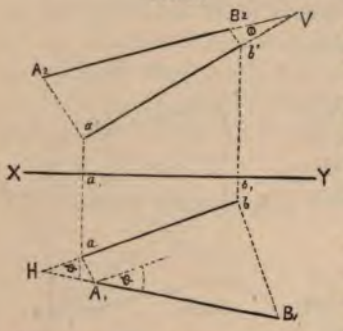


(*a*) *The length of the line.* The projection of a line upon a plane will be less than the line itself, unless the line is parallel to the plane; in the latter case the line and its projection will have the same length.

The orthographic projection of a line can never be greater than the line itself.

A line, its projection on one of the co-ordinate planes, and the projectors from its ends to that plane, form a plane quadrilateral figure, concerning which everything is known for constructing it if the plan and elevation of the line are given. One method, therefore, of finding the length of the line will be to construct this quadrilateral.

FIG. 15.



Thus, let the plan and elevation of a line be given as in fig. 15. Referring to the perspective figure, it will be seen that the line A B, its plan $a\,b$, and the projectors A a, B b form a quadrilateral, the base, $a\,b$, of which is given. Also $A\,a = a'\,a_1$, and $B\,b = b'\,b_1$. Also A a and B b are at right angles to $a\,b$. Hence, to find the length of A B, draw (fig. 15) $a\,A_1$ at right angles to $a\,b$ and equal to $a'\,a_1$; also draw $b\,B_1$ at right angles to $a\,b$ and equal to $b'\,b_1$: $A_1\,B_1$ will be the length required.

Note. If the extremities of the line A B were on opposite sides of the horizontal plane, the perpendiculars $a\,A_1$ and $b\,B_1$ would be drawn on opposite sides of the plan $a\,b$.

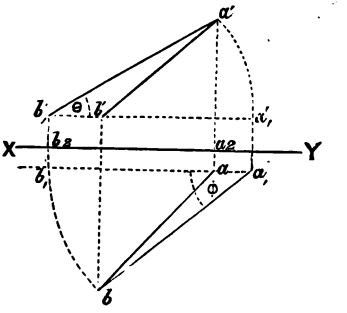
(*b*) *The inclinations of the line to the co-ordinate planes.* Referring to figs. 14 and 15 it is clear that the inclination of A B to the horizontal plane is the angle between $a\,b$ and $A_1\,B_1$, so that the construction for determining the length of a line also serves for finding its inclination. If $a\,b$ and $A_1\,B_1$ do not meet within a convenient distance, the angle between them may be found by drawing through a point in one of them a line parallel to the other; then the angle between these two intersecting lines is the angle required.

The inclination of the line to the vertical plane is found by constructing the quadrilateral $a'\,b'\,B_2\,A_2$, where $a'\,A_2$ and

b' B_2 are at right angles to a' b' and equal to $a\,a_1$ and $b\,b_1$ respectively.

The inclination of a line to the horizontal plane is usually denoted by the Greek letter θ (theta) and its inclination to the vertical plane by the Greek letter ϕ (phi). (Notice that θ is the letter o with a *horizontal* line drawn through it, while ϕ is the same letter with a *vertical* line drawn through it.)

FIG. 16.



The length of the line A B and its inclinations to the coordinate planes may also be found as follows.

Through a draw $a\,b_1$ parallel to X Y. With centre a and, radius $a\,b$, describe the arc $b\,b_1$, cutting $a\,b_1$ at b_1. Draw $b_1\,b'_1$ perpendicular to X Y, meeting a line through b' parallel to X Y at b'_1. $a'\,b'_1$ will be the length of A B, and the angle $a'\,b'_1\,b'$ will be its inclination to the horizontal plane.

The construction for finding the inclination of the line to the vertical plane will be understood from an inspection of the figure.

Comparing this construction with the previous one, it will be seen that in both cases a quadrilateral is drawn having a base equal to one of the projections of the line, and in the first construction the base is made to coincide with that projection, while in the second construction the base is made to lie on X Y.

(*c*) *The traces of the line.* After making the construction for finding the true length of the line as in fig. 15, if A_1 B_1 be produced to meet the plan $a\,b$ at H, then H will be the horizontal trace of the line A B. In like manner if A_2 B_2 be produced to meet the elevation $a'b'$ at V, this point will be the vertical trace of A B. The correctness of this construction is apparent from an inspection of the perspective figure.

If it is only the traces of the line which are required, then

the following construction is simpler than that just given. Produce, if necessary, the elevation to meet X Y at h', through h' draw h' H perpendicular to X Y to meet the plan $a\,b$, or the plan $a\,b$ produced at H; H is the horizontal trace of A B. For, since the horizontal trace of a line is the point where the line meets the horizontal plane, it is a point in the line, and also a point in the horizontal plane, therefore its elevation will be in the elevation of the line and also in X Y, while its plan will be in the plan of the line and in the line through its elevation perpendicular to X Y; but the plan of a point in the horizontal plane is the point itself.

FIG. 17.

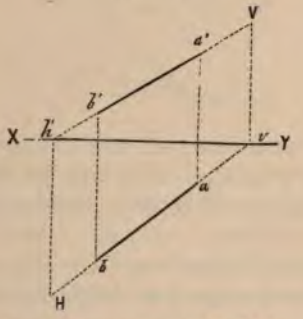
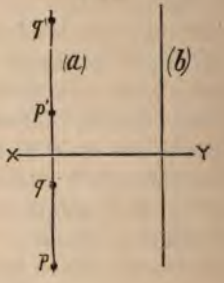


FIG. 18.



In like manner to find the vertical trace produce the plan, if necessary, to meet X Y at v, through v draw v V perpendicular to X Y to meet the elevation, or elevation produced at V; V is the vertical trace of A B.

This construction fails when the plan and elevation of the line are in the same straight line perpendicular to X Y, but the other construction will apply provided the plans and elevations of two points in the line are carefully marked as in fig. 18 (*a*). If the plan and elevation are unlettered as in fig. 18 (*b*), the true length, inclinations, or traces of the line cannot be determined.

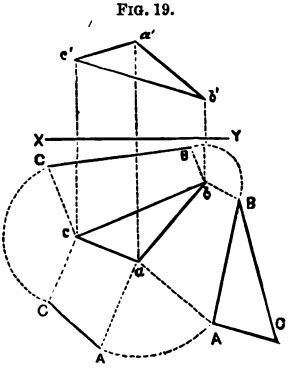
When the projections of a line are perpendicular to X Y, the line itself is perpendicular to X Y, and its inclinations to the co-ordinate planes are complementary.

PROBLEM 6.

From the projections of a plane figure to determine its true form.

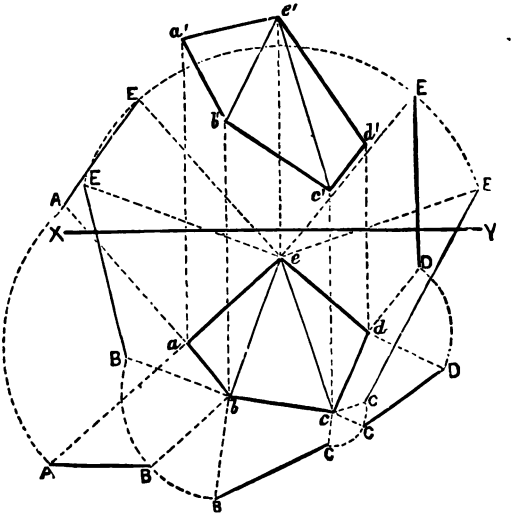
The projection of a plane figure upon a plane will always be different from that figure, excepting when it is parallel to the plane, when the figure and its projection will be exactly alike. To determine the true form of any plane figure it is necessary to know the true distances of a sufficient number of points in it from one another; now these distances can be determined from the projections of the figure by Problem 5.

FIG. 19.



EXAMPLE 1. *Given the plan a b c and the elevation a'b'c' of a triangle; to find its true form.*

FIG. 20.

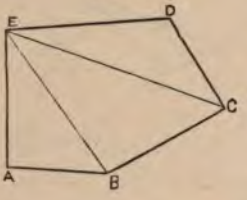


Find the true lengths of the sides of the triangle by Problem 5, and then draw a triangle A B C, having its sides equal to these lengths; A B C will be the true form of the triangle. The construction is shown in the figure.

EXAMPLE 2. *Given the plan a b c d e, and the elevation a'b'c'd'e' of a plane polygon (fig. 20); to find its true form.*

Join *b e*, *b'e'*, *c e* and *c'e'*, determine the true form of each of the triangles A B E, B C E, and C D E, and place them together as shown (fig. 20*a*), so as to form the polygon A B C D E; this is the figure required.

FIG. 20*a*.



Note that when the figure has more than three sides, it is not enough to determine the true lengths of these sides in order to find its true form; the true lengths of a number of its diagonals must also be found.

Note. The traces and inclinations of the sides of the figure can also be found by Problem 5; and it will be found that all the horizontal traces will lie in the same straight line; also all the vertical traces will lie in one straight, and the former line will either meet the latter line at a point on X Y, or they will both be parallel to X Y. This will be referred to in considering the straight line and plane.

PROBLEM 7.

To mark off a given length on a given line.

Let *a b a'b'* be the projections of the line, it is required to find the projections of a point C in this line, so that A C shall be a given length.

Find the true length A B of the line by Problem 5, make A C equal to the required length, through C draw C *c* perpendicular to *a b*, through *c* draw *c c'* perpendicular to X Y

meeting $a'b'$ at c'. c is the plan and c' is the elevation required.

If the plan and elevation of the line are perpendicular to XY, find the plan c as before, and make the distance of c' from XY equal to C c.

The correctness of the construction will be evident in each case if it be worked backwards. After finding c and c', pro-

FIG. 21.

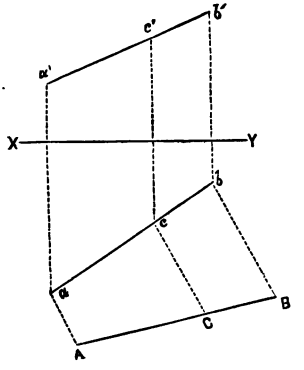
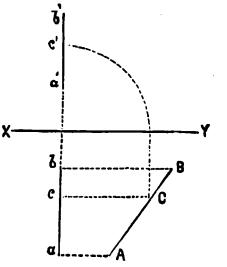


FIG. 22.



ceed to find the true length of A C when it will be found equal to the given distance.

EXERCISES.

1. Show the projections of the following points :—

A $\frac{1}{2}''$ in front of the vertical plane and 2″ above the horizontal plane.

B 2″ „ „ „ „ 2″ below the horizontal plane.

C $1\frac{1}{2}''$ „ „ „ „ in the horizontal plane.

D $1\frac{3}{4}''$ behind „ „ „ $1\frac{1}{2}''$ above the horizontal plane.

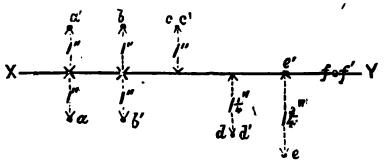
E $1\frac{3}{4}''$ „ „ „ „ $1\frac{1}{2}''$ below the horizontal plane.

N.B.—Use the same ground line for all the projections, and make the perpendiculars from the projections on X Y in each case $\frac{1}{2}''$ apart.

2. Draw the projections of the following points as in Exercise 1.

A 1″ behind the vertical plane and in the horizontal plane.
B in „ „ „ „ „ „
C 1″ in front of the „ „ $1\frac{1}{2}''$ below „ „
D in „ „ „ 1″ „ „ „
E 2″ in front of „ „ „ $1\frac{3}{4}''$ above „ „

FIG. 23.



3. State the exact positions of the points whose projections are given with reference to the co-ordinate planes.

4. A point $1\frac{1}{2}''$ below X Y is the elevation of two points A and B. The plan of A is 2″ above and the plan of B 2″ below X Y. Draw the projections of A and B, and state their positions with reference to the planes of projection.

5. A point 2″ above X Y is the plan of three points A B C. A is 1″ above, B is $1\frac{1}{2}''$ below, and C is in the horizontal plane. Determine the projections of A B and C.

6. Determine the projections of the points A B C D E when
the plan of A is $1\frac{1}{2}''$ below X Y, and the point A is 2″ above the horizontal plane.
„ B „ $1\frac{1}{4}''$ „ „ „ B is $1\frac{1}{2}''$ below the horizontal plane.
„ C „ $1\frac{3}{4}''$ above „ „ C is $1\frac{3}{4}''$ above the horizontal plane.
The elevation of D is 1″ below „ „ D is 2″ behind the vertical plane.
„ E „ in „ „ E is $2\frac{1}{4}''$ behind the vertical plane.

7. Determine the plan and elevation of each of the points A B C D E when

a'	is $1\frac{1}{2}''$	above	X Y,	and the point A is $1''$ in front of the vertical plane.
b'	„ $1\frac{3}{4}''$	below	„	„ B „ $2''$ in front of the vertical plane.
c'	„	in	„	„ C „ $1\frac{3}{4}''$ in front of the vertical plane.
d	„	„	„	„ D „ in the horizontal plane.
e	„ $1''$	above	„	„ E „ $1\frac{1}{2}''$ below the horizontal plane.

8. A line parallel to X Y and $1\frac{1}{4}''$ below it is the plan of a line; two points a, b, in this plan are $2\frac{1}{2}''$ apart. A is $2''$ and B $1''$ above the horizontal plan. Draw the projections of the line.

9. $a\,b$, the plan of a line, makes an angle of 30° with X Y, $a\,b$ being $2\frac{1}{4}''$. The point B is in the horizontal plane, and the point A is in the vertical plane, and $2''$ above the horizontal plane. Show the plan and elevation of the line.

10. The elevation $a'b'$ of a line is inclined at 45° to X Y, $a'b'$ being $2''$ long. a' is $1''$ above X Y; b' is also above X Y. A is $\frac{1}{2}''$ in front and B $2''$ behind the vertical plane. Draw the plan and elevation of A B.

11. The plan of a certain triangle is an equilateral one, $a\,b\,c$ of $2''$ side, one side $a\,b$ coinciding with X Y. The heights of A, B, and C above the horizontal plane are $1''$, $1\frac{1}{2}''$, and $2\frac{1}{4}''$ respectively. Draw the elevation of the triangle.

12. A regular hexagon $a'b'c'd'e'f'$ of $1''$ side is the elevation of a certain figure which is in front of the vertical plane and above the horizontal plane; $a'b'$ is parallel to X Y. The distances of the points A B C D E F from the vertical plane are $1''$, $1''$, $\frac{1}{2}''$, $0''$, $2\frac{1}{2}''$, $1\frac{1}{2}''$ respectively. Draw the plan of the figure.

13. A horizontal line $3''$ long is $1\frac{1}{2}''$ above the horizontal plane, one end of the line is in the vertical plane and the other end $1\frac{1}{4}''$ in front of it. Draw the projections of the line.

14. A vertical line A B 2″ long is 1″ in front of the vertical plane, the lower end B being in the horizontal plane. Draw its plan and elevation.

15. Show the real distance between the points whose projections are given in fig. 24.

16. The plan $a\,b$ of a line is 3″ long, A is 1″ and B is $2\frac{1}{2}$″ above the horizontal plane. What is the true length of A B and its inclination to the horizontal plane?

17. a' and c' (fig. 25) are the elevations of two points in the vertical plane; b is the plan of a point in the horizontal

FIG. 24.

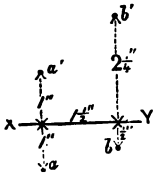
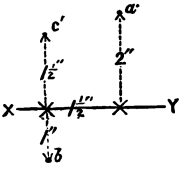


FIG. 25.



plane. Determine the real distance between A and B, B and C, C and A.

18. One end of a rope is fastened to the top of a vertical pole 24′ high; the other end is fixed to the ground at a point 30′ from the pole. Determine the length of the rope. Scale 1″ = 10′.

19. One end of a line is ·75″ below the horizontal plane, the other being 1·5″ above it. The length of its plan is 3″, and it makes 30° with X Y. Show the projections of the line and determine its true length and inclination.

20. The plan of a line is 2″ long, and its elevation is 3″; the projectors of its extremities are 1″ apart measured along X Y. What is its true length and inclination to each of the co-ordinate planes?

21. Define the 'traces of a line.' When will a line (a) have no traces, (b) a horizontal trace only, (c) a vertical trace only, (d) both a horizontal and vertical trace?

22. Assume the projections of a line parallel to neither plane of projection. Determine the traces of the line, and the real distance apart of those traces.

23. What is the true length of a line if its plan measures 2·5″, and if it is inclined at 25° ?

24. A point P is 1·3″ above the horizontal plane, and 1″ in front of the vertical plane; another point Q is ·75″ below the horizontal plane, and 2·25″ behind the vertical plane. The distance between the projectors measured along X Y is 2″. Determine the true length and inclination of the line P Q, and also its traces.

25. $a\,b$, the plan of a line, is 2″ long. The height of A is 1·4″, and of B 2″. Determine and write down the height of the middle point of A B.

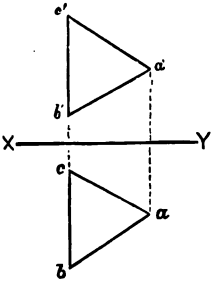
26. The tops of two vertical poles are 45′ apart; the poles are 30′ apart, and the height of one pole is 12 feet. Determine the height of the other pole. Scale 1″ = 10′.

27. Three lines, $a\,o$, $b\,o$, $c\,o$, meeting at o, form the plan of three sticks whose lower ends, A B C, rest on the ground, and whose upper ends meet at O. The height of O is 11′. Determine the lengths of the three sticks. Scale $\frac{1}{4}'' = 1'$. ($a\,o\,b = 125°$, $b\,o\,c = 115°$, $a\,o = 1·2''$, $b\,o = 1·8''$, $c\,o = 1·2''$).

28. The plan of a line 3″ long is 1·5″ in length. At what angle is the line inclined ?

29. Three points form in plan the corners of an equilateral triangle of $2\frac{1}{2}''$ side, and are at heights of 1″, $1\frac{1}{2}''$ and 2″ above the ground. Determine the true form of the triangle of which this is the plan.

FIG. 26.



30. A triangle $a\,b\,c$ ($a\,b = 2''$, $b\,c = 2\frac{1}{4}''$, $c\,a = 2\frac{3}{4}''$) is the plan of one. The height of A is 1″, of B 2″, and of C $2\frac{1}{2}''$. Taking $a\,b$ for the ground line, draw the elevation and determine the true form of the triangle.

31. A regular hexagon $a\,b\,c\,d\,e\,f$ of 1″ side is the plan of a plane figure. The height of A is 1″, of B 1″, of C $1\frac{1}{2}''$, of D 2″, of E 2″, and of F $1\frac{1}{2}''$. Determine the true form of the figure A B C D E F.

32. The equilateral triangles $a\,b\,c$, $a'b'c'$ of 2″ side are the plan and elevation respectively of a triangle; $b\,c$ and $b'c'$ are

perpendicular to X Y, and c and b' are each $\frac{1}{2}''$ from X Y. Determine the true form of the triangle A B C, and the traces of its sides.

33. Draw a square $a'b'c'd'$ of 2″ side, $d'c'$ being on X Y. The sides $a'b'$, $b'c'$, and the diagonal $a'c'$ are the elevations of the sides of a triangle A B C. The points A B and C are at distances of 1″, $1\frac{1}{2}''$ and 2″ respectively in front of the vertical plane. Show the true form of the triangle A B C.

34. Drawing the square as in Exercise 33, determine the true form of the triangle A B C when $a'b'c'$ is also the plan of that triangle.

35. Taking the figure to Exercise 15, show the projections of a point situated in the line joining A and B, and 1″ from A.

CHAPTER III.

SIMPLE SOLIDS IN SIMPLE POSITIONS.

Projection of Solids. It has already been stated (Introduction) that the object of Descriptive Geometry is to convey to the mind an impression of the exact form and size of objects, which have length, breadth, and thickness, by means of representations on a surface, which has length and breadth only.

Now a solid may be conceived as made up of an immense number of small particles or material points, the relative positions of which may be represented by their projections on two planes, as explained in the preceding chapters; but, as in looking at a solid it is generally only those points on its external surface which are seen, all our impressions of the form and size of objects are derived from the form and extent of their surfaces. It is therefore unnecessary in representing an object on paper to give the projections of points in its interior —that is, it is only necessary to represent the surface of the solid.

Again, the form and extent of a surface is generally known

when we know the forms, lengths, and relative positions of a sufficient number of lines on that surface. But we have seen that by the method of projections the forms, lengths, and relative positions of lines may be represented on a single flat surface. Hence, to represent a solid on paper we need to show the projections of a sufficient number of lines on its surface.

When the solid has plane faces, the projections of the boundary lines of those faces are all that is necessary in order to represent them.

DEFINITION OF SOLIDS.

A *polyhedron* is a solid bounded entirely by planes.

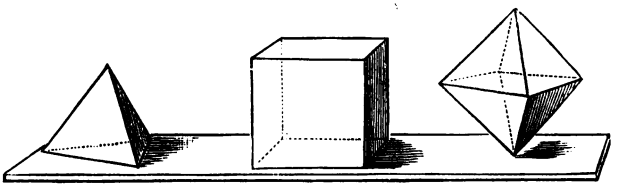
The *edges* of a polyhedron are the lines of intersection of its bounding planes.

The *sides* or *faces* of a polyhedron are the plane figures formed by its edges.

A polyhedron is said to be *regular* when its faces are equal and regular polygons, and each adjacent pair include the same angle.

There are only five regular polyhedra—viz. the *tetrahedron*, the *cube*, the *octahedron*, the *dodecahedron*, and the *icosahedron*.

FIG. 27.



The tetrahedron. The cube. The octahedron.

The *tetrahedron* has four faces, all equilateral triangles.

The *cube* has six faces, all squares.

The *octahedron* has eight faces, all equilateral triangles.

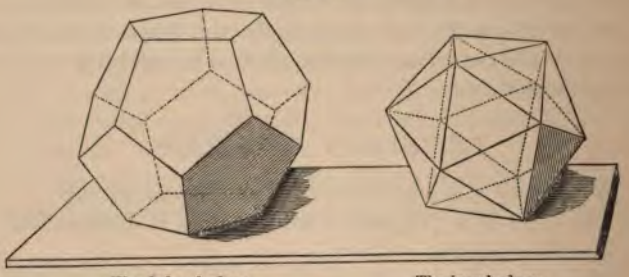
The *dodecahedron* has twelve faces, all pentagons.

The *icosahedron* has twenty faces, all equilateral triangles.

Models of these five solids can readily be made out of cardboard or stiff paper in the following manner: Draw figs. 29

to 33 on the pieces of cardboard or paper to any convenient size, and cut them out along by their boundary lines, and pierce a number of small holes along the thin lines as shown; now fold them up round the thin lines as hinges until the required

FIG. 28.



The dodecahedron. The icosahedron.

solid is formed, then cover the joints with strips of thin gummed or glued paper.

The use of the small holes is to insure that the cardboard or paper will bend along the correct lines and leave sharp edges.

FIG. 29.

FIG. 30.

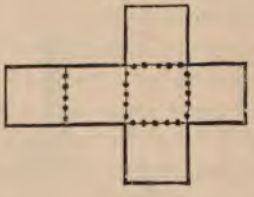


FIG. 31.

The student would do well to make a set of these models for himself.

Casts from the above paper models can easily be got by filling them with 'plaster of Paris' and then removing the paper when the plaster has solidified.

Figs. 29 to 33 are called the *developments* of the surfaces of these solids.

A *prism* is a polyhedron having two of its faces, called its *ends* or *bases*, parallel, and the rest parallelograms.

Those faces of a prism which are parallelograms are generally called its *sides*.

FIG. 32.

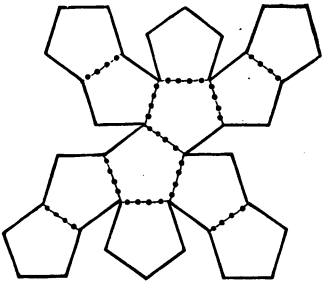
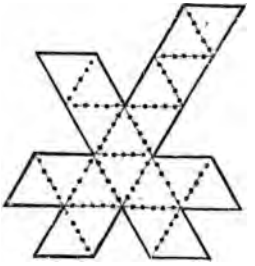


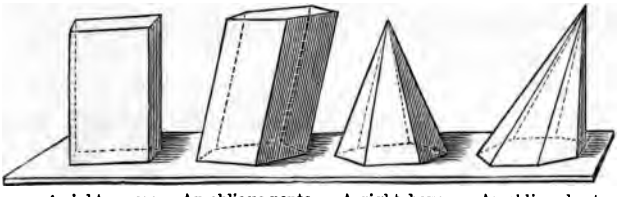
FIG. 33.



A *parallelepiped* is a prism whose bases are parallelograms.

A *pyramid* is a polyhedron having a polygon for its base, and for its sides it has triangles which have a common vertex, and the sides of the polygon for their bases.

FIG. 34.



A right square prism. | An oblique pentagonal prism. | A right hexagonal pyramid. | An oblique heptagonal pyramid.

The common vertex of the triangles is called the *vertex*, or *apex*, of the pyramid.

The *axis* of a prism is the straight line joining the centres of its ends; and the axis of a pyramid is the straight line from its vertex to the centre of its base.

A *right prism* is one having its axis at right angles to its ends.

An *oblique prism* is one having its axis inclined to its ends.

A *right pyramid* has its axis perpendicular to the plane of its base.

An *oblique pyramid* has its axis inclined to the plane of its base.

The *altitude* of a prism is the perpendicular distance between the planes of its ends; and the altitude of a pyramid is the perpendicular distance of its vertex from the plane of its base.

Prisms and pyramids are named from the form of their bases—as triangular, square, pentagonal, hexagonal, &c.

A *cylinder* resembles a prism. If the sides of the bases of a prism be continually diminished in length and increased in number, the ultimate form of the boundary lines of the bases will be curved lines, and the ultimate form of the prism will be a *cylinder*.

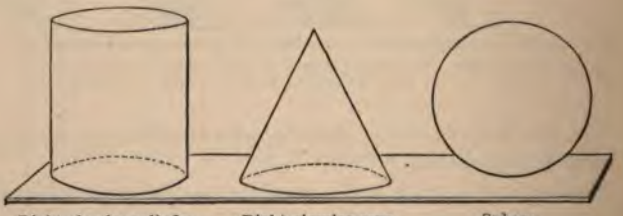
A *right cylinder* has its axis at right angles to its ends.

A *right circular cylinder* has its axis at right angles to its ends, which are equal circles.

A right circular cylinder may also be defined as a solid described by the revolution of a rectangle about one of its sides, which remains fixed.

The fixed line about which the rectangle revolves is the *axis*, and the circles described by the opposite revolving sides are the *bases*, or *ends*.

FIG. 35.



Right circular cylinder. Right circular cone. Sphere.

A *cone* resembles a pyramid. If the sides of the base of a pyramid be continually diminished in length and increased in number, the ultimate form of the boundary line of the base

will be a curved line and the ultimate form of the pyramid will be a *cone*.

A *right circular cone* has its axis at right angles to its base, which is a circle.

A right circular cone may also be defined as a solid, described by the revolution of a right angled triangle about one of the sides containing the right angle, which side remains fixed.

The fixed line about which the triangle revolves is the axis, and the circle described by the other side containing the right angle is the base.

A *sphere* is a solid every point on the surface of which is at the same distance from a point within it, called its centre.

A sphere may also be defined as a solid described by the revolution of a semicircle about its diameter, which remains fixed.

The centre of the semicircle is the centre of the sphere.

PROBLEM 8.

To draw the projections of a prism when one end is parallel to one of the co-ordinate planes, and one side of that end or an adjacent face is inclined at a given angle to the other plane, or to the ground line.

Since the ends of the prism are parallel to one of the co-ordinate planes, their projections on that plane will show their true form, and as the form of the ends is supposed to be given, this projection is therefore drawn first, being so placed with respect to the ground line as to fulfil the second condition of the problem.

EXAMPLE 1. *To draw the projections of a square prism base,* $1\frac{1}{2}''$ *side, altitude* $2\frac{1}{2}''$, *when one end is on the H.P. and one side of that end is inclined at* 30° *to the V.P.*

First draw the plan, which will be a square of $1\frac{1}{2}''$ side, having one side inclined at 30° to the ground line. From the plan the elevation is got by drawing perpendiculars to X Y from the corners of the square, as shown. The height

of the elevation is equal to the altitude of the prism, namely $2\frac{1}{2}''$.

It will be noticed that in the elevation one of the edges is dotted because that edge is behind the solid and therefore hid by it. Great care must be taken by the student to make those lines of the elevation dotted which would be hid by the object when viewed from the front, and those lines of the plan dotted which would be hid by the object when viewed from above.

FIG. 36.

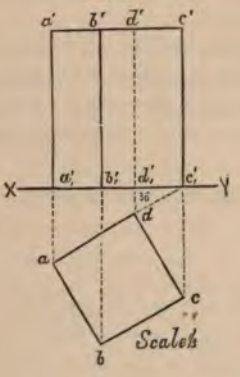
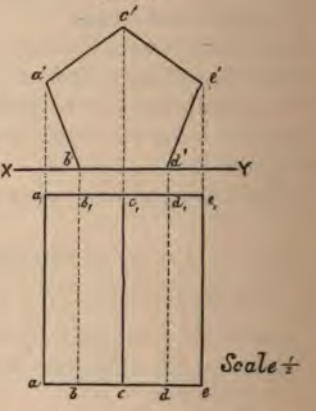


FIG. 37.



EXAMPLE 2. *To draw the plan and elevation of a pentagonal prism when it lies with one side on the H.P. and its ends parallel to the V.P.; side of pentagon* 1″, *length of prism* 2″.

First draw the elevation, which will be a pentagon of 1″ side, having one side on X Y.

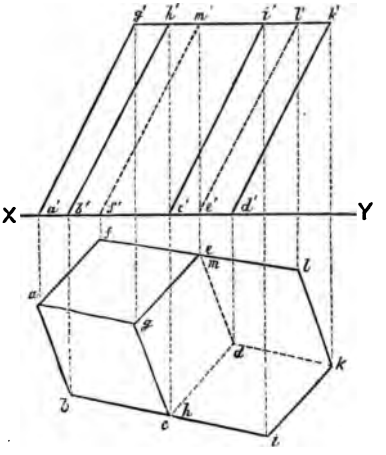
The plan is got from the elevation in the same way as the elevation was got from the plan in Example 1.

EXAMPLE 3. *The bases of an oblique prism are hexagons of* 1″ *side. The perpendicular from the centre of one end on to the plane of the other passes through one of the angular points of the latter. One end of the prism is in the H.P., and the plan of its axis makes* 10° *with the ground line. Draw plan and elevation; altitude of prism* 2″.

The plan of the ends will be two hexagons of 1″ side, with the sides of the one respectively parallel to the sides of the other. The line joining the centres of the hexagons will be the plan of the axis which will be parallel to one side of each; hence the hexagons will have one side inclined at 10° to X Y.

The elevations of the ends will be straight lines, 2″ apart, one being in X Y; and the elevations of the corners will be where the perpendiculars from the plan cut these two lines, as shown in fig. 38.

FIG. 38.



PROBLEM 9.

To draw the projections of a pyramid when its base is parallel to one of the co-ordinate planes, and one side of the base is inclined at a given angle to the other plane or to the ground line.

The projection of the pyramid on the plane to which the base is parallel is drawn first, and will be a polygon equal to the base with lines drawn from the projection of the vertex to the angular points of the polygon.

EXAMPLE 1. *A right pyramid has a pentagon of* 1″ *side for its base, and has an altitude of* 2″. *The base is in the H.P., and one side of it makes* 60° *with X Y. Draw plan and elevation.*

Begin with the plan. Draw a pentagon of 1″ side, having one side inclined at 60° to X Y. Join the centre of this pentagon with its angular points and the plan will be complete.

Since the base is in the horizontal plane the elevations of all its corners will be in X Y, and the elevation of the vertex will be 2″ above it in the line through the centre of the pentagon perpendicular to X Y.

FIG. 39.

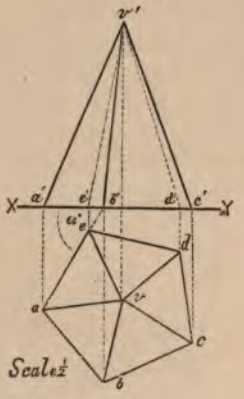
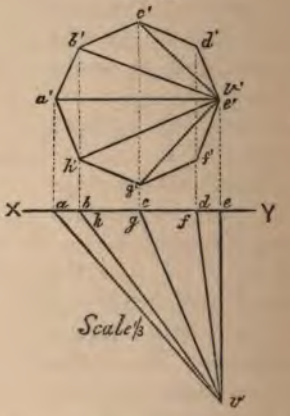


FIG. 40.



EXAMPLE 2. *An oblique octagonal pyramid has its base in the V.P. A perpendicular from the apex to the base passes through one of the angular points of the octagon. The axis is horizontal: side of octagon* 1″, *altitude of pyramid* 2″. *Draw plan and elevation.*

First draw the elevation, the outline of which will be an octagon of 1″ side. A line from the centre of this octagon to one of its angular points will be the elevation of the axis which is to be horizontal, therefore one diameter of the octagon must be parallel to the ground line.

One extremity of the horizontal diameter will be the elevation of the vertex; hence, to complete the elevation, join this point with all the other angular points of the octagon.

The plan of the base will be in X Y since the base is in the V.P., and the plan of the vertex will be 2″ below it in the line through its elevation perpendicular to X Y.

PROBLEM 10.

To determine the altitude of a tetrahedron.

Referring back to the definition of a tetrahedron and to fig. 27, it will be seen that it is a right pyramid on a triangular base. Its projections may therefore be drawn by Problem 9 if its base and altitude are known.

As will readily be understood from the definition of a tetrahedron, its altitude will depend upon the length of edge, and for every size of base there will be one and only one altitude.

FIG. 41.

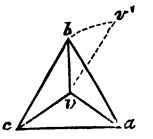


Fig. 41 shows the plan of a tetrahedron when standing with one face on the horizontal plane, and it is required to find the height of the point V. $a\,v$ is the plan of one of the sloping edges, and, since all the faces are equilateral triangles, the true length of the edge of which $a\,v$ is the plan will be equal to $a\,b$. It is therefore clear that if, with a as centre and $a\,b$ as radius, an arc be drawn to cut $v\,v'$, which is perpendicular to $a\,v$ at v', $v\,v'$ will be the required altitude.

PROBLEM 11.

To draw the projections of an octahedron when one axis is perpendicular to one of the co-ordinate planes, and an edge not meeting that axis is inclined at a given angle to the other plane.

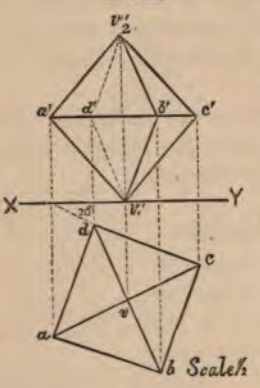
The lines joining the opposite angular points of the octahedron are its axes. There are three of these lines all equal in length, and bisecting one another at right angles at the centre of the solid.

It will be found on examination that the octahedron consists of two square pyramids placed base to base, the triangular faces being equilateral triangles. Two of the axes of the octahedron are the diagonals of the square which

forms the common base of the two forementioned pyramids, while the third axis is the line joining their vertices.

The projection of the octahedron on the plane to which the axis is perpendicular is drawn first, and will be a square, with its diagonals one side making the given angle with the ground line. The method of finding the other projection will best be understood by considering the following example.

FIG. 42.



EXAMPLE. *The edge of the octahedron is* $1\frac{1}{4}''$, *one axis is vertical, and one of the horizontal edges makes* 20° *with the V.P. To draw plan and elevation.*

First draw the plan, which will be a square of $1\frac{1}{4}''$ side, one side being inclined at 20° to X Y.

Find the elevations v_1' v_2' of the extremities of the vertical axis. v_1' v_2' will be equal to $a\,c$ or $b\,d$, and will be in the line through v perpendicular to X Y. Bisect v_1' v_2' by the horizontal line a' c'. Through $a\,b\,c$ and d draw perpendiculars to meet a' c' at a' b' c' d'; complete the elevation by joining each of the latter points with v_1' and v_2'.

PROBLEM 12.

To draw the projections of a right circular cylinder when its axis is at right angles to one, or parallel to both, of the co-ordinate planes.

When the axis is perpendicular to one of the co-ordinate planes, the projection on that plane will be a circle equal to that of the ends of the cylinder. The projection on the other plane will be a rectangle, one side being parallel to the ground line and equal to the diameter of the circle, while the other will be equal in length to the axis of the cylinder.

When the axis is parallel to both planes of projection, each projection will be a rectangle equal to the one just described, but that side which is equal to the diameter of the cylinder will be perpendicular to the ground line.

PROBLEM 13.

To draw the projections of a right circular cone when its axis is at right angles to one, or parallel to both, of the co-ordinate planes.

When the axis is perpendicular to one of the co-ordinate planes, the projection on that plane will be a circle equal to that of the base of the cone. The projection on the other plane will be an isosceles triangle, its base being parallel to the ground line and equal to the diameter of the base of the cone, and having an altitude equal to that of the cone.

When the axis is parallel to both planes of projection, each projection will be a triangle equal to the one just described, but the base will be perpendicular to the ground line.

PROBLEM 14.

To draw the projections of a sphere.

All projections of a sphere are circles, having a diameter equal to that of the sphere.

The plan and elevation will have their centres in the same straight line perpendicular to the ground line.

EXERCISES.

1. Draw the projections of a cube of $1\frac{1}{2}''$ edge when in each of the following positions:—

(*a*) One face in the H.P. and an adjacent face in the V.P.

(*b*) One face in the V.P. and an adjacent face inclined at 35° to the H.P., the lower edge of the latter face being in the H.P.

(*c*) When the edge nearest to the V.P. is vertical and $\frac{1}{2}''$ in front of the V.P., the lower end of that edge being $\frac{1}{4}''$ above the H.P., and a face containing it inclined at 20° to the V.P.

2. The ends of a right prism are regular hexagons of 1″ side, and the prism is 2″ long. Draw its plan and elevation in each of the following positions:—

(*a*) When one end is in the H.P. and one side parallel to the V.P., and ·2″ in front of it.

(*b*) When one end is in the V.P., the lowest edge ·5″ above the H.P., and a side containing that edge inclined at 30° to the H.P.

(*c*) When situated as in (*b*), excepting that the side is inclined at 45° instead of 30°.

3. Draw the plan of a right heptagonal prism (base 1″ side, axis 2″ long) when lying with one side in the H.P.

4. An oblique prism, whose bases are squares of 1″ side, stands with one of them in the H.P. The square, which is the plan of the upper end, has one angular point coinciding with the centre of the square, which is the plan of the other end. The altitude of the prism is $1\frac{3}{4}$″. Draw a plan and elevation of the prism when in each of the following positions:—

(*a*) One diagonal of the base perpendicular to X Y.

(*b*) The other diagonal of the base perpendicular to X Y.

(*c*) One side of the base perpendicular to X Y.

5. A right pyramid has a square of 1·5″ side for its base, and an altitude of 2″. Draw its plan and elevation when its base is in the H.P., and one side of that base is inclined at 30° to X Y.

6. An octagon of 1″ side is the base of a right pyramid, whose axis is 2·25″ long. Draw its plan when the base is in the V.P., and the lowest corner of the base is ·25″ below the next.

7. Draw the plan and elevation of a tetrahedron of 1·5″ edge when one face is in the H.P. and an edge is inclined at 20° to the V.P.

8. Draw the projections of an octahedron of 1·5″ edge when one axis is vertical and one edge inclined at 40° to the V.P.

9. The base of a pyramid standing on the H.P. is a rhombus of 2″ side, with an acute angle of 60°. The vertex *is vertically* over the middle point of one of the sides of the

base, and 2·5″ distant from it. Draw plan and elevation when a side of the base adjacent to that under the vertex is parallel to X Y. Also determine the true length of each of the sloping edges.

10. Show the projections of a right circular cylinder 1·5″ diameter and 2″ long when in each of the following positions:—

(*a*) Axis vertical.

(*b*) Axis at right angles to the V.P.

(*c*) Axis parallel to both planes of projection and 1″ distant from each.

11. Draw plan and elevation of a right circular cone, base 2″ diameter, axis 2″ long, (*a*) when the base is in the V.P.; (*b*) when the axis is vertical.

12. Draw the projections of a sphere 2″ diameter (*a*) when its centre is 1·5″ above the H.P. and 1·25″ in front of the V.P.; (*b*) when its centre is in the ground line.

13. The cylindrical part of a bolt is 1″ diameter and 2″ long, and the head is a hexagonal prism (base 1″ side, axis 1″ long). The axes of prism and cylinder are in the same straight line. Draw a plan and elevation when the bolt stands on its head, and has one side of that head parallel to the V.P.

14. A hexagonal prism (side of base 1″, height 1″), having its base in the H.P., and one side inclined at 10° to the V.P., is surmounted by a tetrahedron having the corners of its base at three of the angular points of the prism, one corner coinciding with that corner of the prism which is nearest to the V.P. Draw plan and elevation.

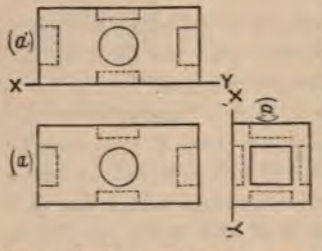
15. A rectangular slab, $2\frac{1}{2}'' \times 1\frac{1}{2}'' \times \frac{1}{2}''$, rests with one of its large faces on the H.P. Four square prisms, $\frac{1}{4}'' \times \frac{1}{4}'' \times 1''$, stand on the upper face of this slab, having one angle of each base coinciding with an angle of that face. On the top of these prisms rests a right pyramid having a base equal to the large faces of the slab, which base is vertically over the slab, and having an altitude of 1″. Draw a plan and elevation of the whole when the long sides of the slab are inclined at 25° *to the V.P.*

CHAPTER IV.

CHANGING THE PLANES OF PROJECTION.

Auxiliary Projections. It has been already stated that one plan and elevation of an object determines its true form, but there are cases where this is not quite true, and many others where additional projections would tend very much to give a clearer understanding of the object. Thus, in fig. 43, we have a plan (a) and elevation (a') of a rectangular block having recesses in all its faces. It is clear from the given plan and elevation that the recesses in the top, bottom, front, and back are cylindrical, but those in the other two faces might be either square or circular in their cross section. To settle the form of the end holes we would require an end view (a'_1)—that is, a projection on a plane parallel to the ends. This second vertical plane will intersect the horizontal plane in a straight line, which will be another ground line.

FIG. 43.



The end elevation is projected from the plan in the same way as the front elevation.

PROBLEM 15.

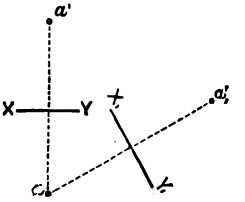
A plan and elevation of a point being given, to find another elevation from them.

The angle between two vertical planes is the same as the angle between their ground lines. The ground lines, therefore, fix the position of the vertical planes.

Let $a\,a'$ be the projections of a point A, and $X_1\,Y_1$ the

ground line of a second vertical plane. The projection of the point A on the second vertical plane will be in a straight line through a at right angles to $X_1 Y_1$; and since the elevation of a point gives its distance from the horizontal plane, the new elevation a'_1 will be at a distance from $X_1 Y_1$ equal to the distance of a' from X Y.

FIG. 44.



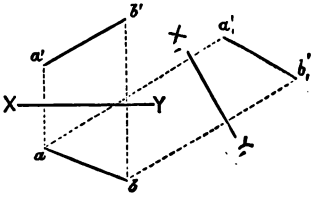
If the first elevation is below the first ground line, the second elevation will be at the same distance below the second ground line.

PROBLEM 16.

A plan and elevation of a line being given, to find another elevation from them.

Let $a\,b$, $a'\,b'$ be the plan and elevation of a line, A B, and $X_1 Y_1$ the ground line of the new vertical plane. Find the

FIG. 45.



new elevations a'_1, b'_1 of the points A and B by the preceding problem; the line $a'_1\,b'_1$ will be the elevation required.

PROBLEM 17.

Given the projections of a point, to determine its distance from the ground line.

The distance of a point from the ground line is the length of the perpendicular from the point on to the ground line. As in general this perpendicular will be inclined to both

co-ordinate planes, neither its plan nor elevation will show its true length; but by making an end view of the co-ordinate planes and the perpendicular, the latter will be projected on a plane parallel to it, and will therefore have its true length shown.

EXAMPLE. *a a'* (fig. 46) *are the projections of a point A; to find the distance of A from X Y.*

Draw $X_1 Y_1$ at right angles to X Y. Draw V.P. in the same straight line as X Y. $X_1 Y_1$ and V.P. are the end eleva-

FIG. 46.

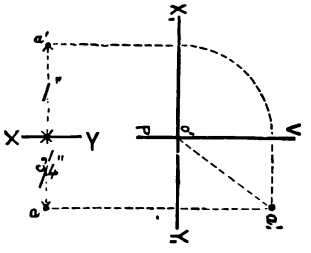
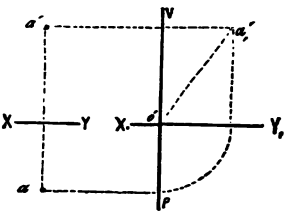


FIG. 47.



tions of the horizontal and vertical planes respectively, and the point *o'* is the end elevation of X Y. Determine a'_1, the end elevation of A, as in Problem 15. $a'_1\ o'$ is the distance required.

Very often the end elevation is drawn as shown in fig. 47, which is just that of fig. 46, turned through 90°.

PROBLEM 18.

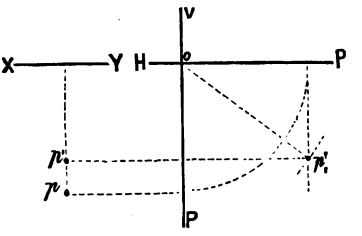
To determine the projections of a point having given its distance from one of the co-ordinate planes and its distance from the ground line.

First make an end view of the co-ordinate planes, then, referring to fig. 47, the point a'_1 has to be found from the condition that it is at a given distance from $X_1 Y_1$ or V.P., and also at a given distance from *o'*. Having found a'_1, the construction for finding *a* and *a'* is evident, being just the construction of Problem 17 worked backwards.

EXAMPLE. *A point, P, which is below the horizontal plane, is 2″ in front of the vertical plane and 2½″ from the ground line; show its plan and elevation.*

Two lines, H.P. and V.P., at right angles to one another will be the end view of the co-ordinate planes. As the point is 2″ in front of the vertical plane, its end elevation will be 2″ from the line V.P.; draw therefore a line parallel to V.P., at a distance of 2″, this line will contain the end elevation. Since the point is 2½″ from the ground line, its end elevation will lie on a circle of 2½″ radius, having O for its centre. The point p'_1, where the circle cuts the parallel line (below H.P.), is the end elevation of the point P. From this the plan p and elevation p' are found by the construction shown in the figure.

FIG. 48.



PROBLEM 19.

A plan and elevation of a plane figure being given, to draw another elevation from them.

This problem is just the application of Problems 15 and 16.

EXAMPLE 1. *The squares a c, a'c' (fig. 49), of 1¼″ side, are the plan and elevation respectively of a rectangle; to draw another elevation, the new ground line making an angle of 53° with the first.*

Find the new elevations of the points A B C and D by Problem 15, and join them as shown in fig. 49.

EXAMPLE 2. *A circle 1½″ in diameter has its plane vertical; to show an elevation on a vertical plane inclined at 60° to the plane of the circle.*

First draw an elevation on a plane parallel to the plane of

the circle; this elevation will be a circle of $1\frac{1}{2}''$ diameter. The plan will be a straight line parallel to X Y. Now draw a new ground line, X_1 Y_1, inclined at 60° to X Y. Take a convenient number of points 0 1 2 3 . . . on the circumference

FIG. 49.

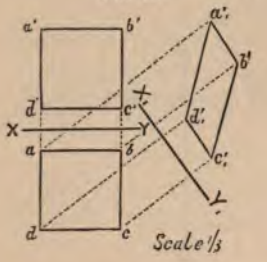
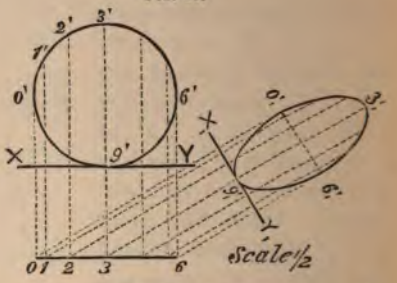


FIG. 50.



of the circle, and find their elevations $0'_1$ $1'_1$ $2'_1$ $3'_1$ . . . on the plane of which X_1 Y_1 is the ground line by Problem 15, join these points by a 'fair curve,' and the elevation required is complete.

Note. The projection of a circle is an equal circle, an ellipse, or a straight line according as the plane of the circle is parallel, inclined, or perpendicular to the plane of projection. When the projection is an ellipse the major axis is equal to the diameter of the circle.

In fig. 50, $0'_1$ $6'_1$ is the minor axis, and $3'_1$ $9'_1$ is the major axis.

After finding the axes of the ellipse, the curve may be put in by any of the methods for drawing ellipses.

PROBLEM 20.

From one plan and one elevation of a solid to draw another elevation.

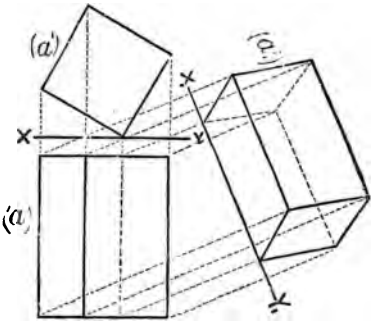
This problem, like the last, is just an application of Problems 15 and 16.

EXAMPLE 1. *A square prism, side of base* 1″, *axis* $1\frac{3}{4}''$, *has one long edge in the horizontal plane, and a face containing that*

edge inclined at 30°; *to draw its plan and an elevation on a vertical plane inclined at* 70° *to the ends of the prism.*

First draw the elevation (a') and plan (a) by Problem 8. Draw the new ground line to make 70° with the plans of

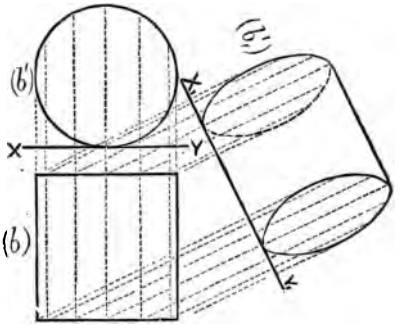
FIG. 51.



the ends. The construction for finding the new elevation (a'_1) will be easily understood from the figure.

EXAMPLE 2. *A cylinder,* $1\frac{1}{2}''$ *diameter, and* $1\frac{1}{2}''$ *long, has its axis horizontal; to draw an elevation on a plane inclined at* 65° *to the ends.*

FIG. 52.



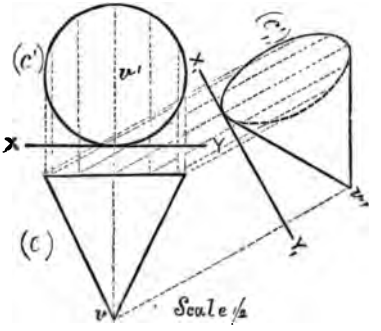
First draw the elevation (b') and plan (b). Draw the new ground line inclined at 65° to the first ground line.

Determine the ellipses which are the new elevations of the ends in the same way as in Example 2, Problem 19. Parallel tangents to these ellipses complete the required elevation.

EXAMPLE 3. *A cone, base* $1\frac{1}{2}''$ *diameter, axis* $1\frac{1}{2}''$ *long, has its axis horizontal; to draw an elevation on a plane inclined at* 60° *to the base.*

First draw the elevation (c') and plan (c). Draw $X_1 Y_1$ inclined at 60° to X Y. Next determine the ellipse which is

FIG. 53.



the new elevation of the base as in Exercise 2, Problem 19. Tangents through v'_1, the new elevation of the apex, to the ellipse complete the required elevation.

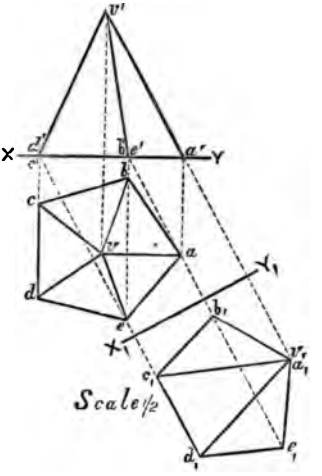
PROBLEM 21.

To draw the plan of a solid when a given line in it is vertical.

First draw a plan and elevation of the solid in a simple position so that the given line is parallel to the vertical plane, that is, so that the plan of this line is parallel to the ground line. Make the new ground line perpendicular to the elevation of the given line, and project the new plan from the elevation in a manner similar to that for projecting new *elevations,* as in the preceding problems of this chapter.

EXAMPLE 1. *To draw the plan of a pentagonal pyramid when one edge passing through the vertex is vertical. Side of base* 1″, *altitude* 1½″.

FIG. 54.



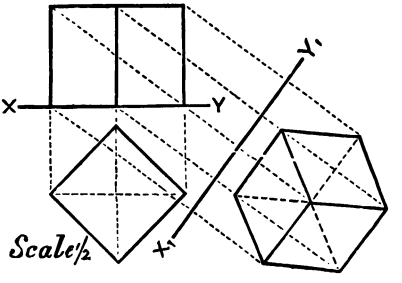
Draw first a plan and elevation of the pyramid when standing with its base on the horizontal plane, the plan $v\,a$ of a sloping edge being parallel to X Y.

Draw X_1Y_1 perpendicular to $v'a'$. Through $a'b'c'd'e'$ and v' draw perpendiculars to X_1Y_1, and mark off on these from X_1Y_1 lengths equal to the distances of $a\,b\,c\,d\,e$ and v from X Y respectively. This determines the points $a_1b_1c_1d_1e_1v_1$; joining these as shown we get the required plan.

EXAMPLE 2. *To draw the plan of a cube when one of its diagonals is vertical. Edge of cube* 1″.

Commence by drawing a plan and elevation of the cube when one face is on the horizontal plane. As the diagonal of the square which is the plan of the cube when in this position is the plan of the diagonal of the solid, the plan will have a diagonal parallel to the ground line. Make the new ground line perpendicular to the elevation of the diagonal, and deduce the required plan as in the preceding example. The outline of this plan ought to be a regular hexagon.

FIG. 55.



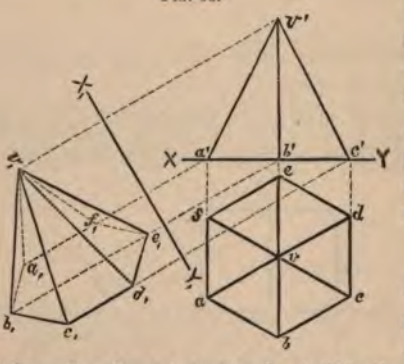
Note. The plan of a solid may be drawn when a given line in it is inclined at *any* given angle to the horizontal plane by proceeding in the manner just explained, but making the new ground line inclined at the given angle to the elevation of the given line instead of perpendicular. But as the solid in this case can occupy any number of positions with reference to the horizontal plane and still fulfil the given condition, the problem is indefinite. In the case where the given line is vertical, however, the plan is always the same. These remarks will be understood if the student imagines the solid to turn round the given line as an axis.

PROBLEM 22.

To draw the plan of a solid when one face is inclined at a given angle, the base of that face being horizontal.

Commence by drawing a plan and elevation of the solid in such a position that the elevation of the given face is a straight line, and the plan of its base is perpendicular to the ground line. Next draw a new ground line inclined to the elevation of the given face at the given angle, and deduce the required plan as in Problem 21.

FIG. 56.



EXAMPLE. *To draw the plan of a hexagonal pyramid, base* $1\frac{1}{4}''$, *altitude* $2\frac{1}{4}''$, *when one of its triangular faces is inclined at* 60°, *the base of that face being horizontal.*

First draw a plan and elevation of the pyramid when its base is on the horizontal plane, one side of the base being perpendicular to the ground line. In this position one of the triangular faces will have for its elevation a straight line.

Draw X_1Y_1 inclined at 60° to $v'a'$, and find the new plan as in Problem 21.

Note. The student should remember the following laws which are applied in solving the problems of this chapter:—

(1) The plan and elevation of a point are, in the same straight line, perpendicular to the ground line.

(2) When a number of elevations are projected from the same plan, the distances of all the elevations of any point from their corresponding ground lines are the same.

(3) When a number of plans are projected from the same elevation, the distances of all the plans of any point from their corresponding ground lines are the same.

EXERCISES.

1. Draw an elevation of the points given in Exercise 3, Chapter II., on a ground line passing through a and e'.

N.B. In fig. 23 the perpendiculars from the projections on X Y are $\frac{1}{2}''$ apart.

2. Determine the elevation of the line given in Exercise 8, Chapter II., on a ground line perpendicular to X Y, and passing through b.

3. Determine the distance from the ground line of each of the points given in Exercise 1, Chapter II.

4. Determine the distance of each of the following points from the ground line:—

A is $1\frac{1}{2}''$ behind the V.P., and $1\frac{1}{4}''$ below the H.P.

B is $1''$ in front of the V.P., and $1''$ below the H.P.

c is $2''$ above X Y, and C is $2''$ above the H.P.

d' is $1\frac{3}{4}''$ below X Y, and D is $1\frac{3}{4}''$ behind the V.P.

E is in the H.P., and $1''$ behind the V.P.

5. Show the projections of the following points:—

A $2''$ in front of the V.P., above the H.P., and $3''$ from X Y.

B $1''$ above the H.P., behind the V.P., and $2''$ from X Y.

C $1\frac{1}{2}''$ behind the V.P., below the H.P., and $2''$ from X Y.

D $2\frac{1}{4}''$ below the H.P., in front of the V.P., and $3''$ from X Y.

E *in the V.P., above the* H.P., and $1\frac{1}{2}''$ from X Y.

6. Show the elevation of the triangle given in Exercise 33, Chapter II., when the ground line coincides with $c\,a$.

7. Draw the plan and elevation of the figure as given in Exercise 12, Chapter II., and a new elevation on a V.P. perpendicular to the first V.P.

8. A right prism whose ends are hexagons of 1·25″ side, and whose axis is 3·25″ long, lies with one side on the horizontal plane. Draw its plan, and give an elevation on a ground line which makes an angle of 30° with the long side of the plan.

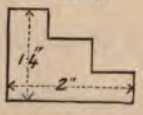
9. A rectangle $2\frac{1}{2}'' \times 2''$, with a square of 1″ side, whose centre coincides with the centre of the rectangle, and whose sides are parallel to those of the rectangle, is the front elevation of a right hollow prism $\frac{3}{4}''$ thick, resting on the H.P. Draw an elevation of it on a V.P. inclined at 40° to the planes of the larger faces.

10. A rectangle $3'' \times 2''$, with the line joining the middle points of its short sides, is the plan of a right triangular prism resting on the ground. Draw an elevation on a ground line, making an angle of 60° with the short sides of the rectangle.

N.B. The triangle is equilateral.

11. The end elevation of three equal steps is given. The length of the plan is $2\frac{3}{4}''$. Draw an elevation on a V.P. inclined at 35° to the front of the steps.

FIG. 57.



12. The cross section of a hollow prism is a pentagon of $1\frac{1}{4}''$ side, with a circle 1″ diameter, having its centre at the centre of the pentagon. The solid, which is $2\frac{1}{2}''$ long, lies with its long edges horizontal, and one of its sides inclined at 25°. Draw an elevation of it on a V.P. which makes an angle of 65° with the planes of its ends.

13. The cross section of a hollow prism is a regular hexagon of $1\frac{1}{2}''$ side, with another concentric hexagon of 1″ side, the sides of the latter being respectively parallel to those of the former. Draw an elevation of this solid when one of its rectangular faces is inclined at 15° to the H.P., and the axis horizontal but inclined at 60° to the V.P. Prism 3″ long.

14. Draw the plan of a square prism when a diagonal of *the solid is* vertical. Side of base $1\frac{1}{2}''$, axis 2″.

15. A square of 1·5″ side, having one side *a b* inclined at 0° to X Y, is the plan of a square prism resting on the H P. The height of the prism is ·5″. Suppose it to be tilted about the edge *a b* through an angle of 50°, and draw the plan and an elevation on X Y.

16. Draw the ellipse which is the plan of a circle $2\frac{1}{2}$″ diameter when inclined at 40° to the H.P., and an elevation on a ground line inclined at 30° to the major axis of the ellipse.

17. Draw the plan of a cylinder (diam. 2″, axis 3″) when its axis is inclined at 45° to the H.P., and an elevation on a ground line making an angle of 40° with the plan of the axis.

18. A cylinder 3″ diameter and 3″ long has its axis inclined at such an angle that the plans of the two ends touch each other; represent it in this position. An elevation to be made on a ground line, making an angle of 60° with the plan of the axis.

19. Draw the plan of an octahedron of 2″ edge resting with one face on the H.P., and add an elevation on a ground line not parallel to any side of the plan.

20. A pyramid 3″ high has a pentagon A B C D E of 2″ side for its base. V being the vertex, draw the plan of this solid—

(*a*) When the two edges B V, C V are horizontal.

(*b*) When the edge E V is vertical.

Both plans to be projected from the same elevation.

21. A hexagon of 1·5″ side is the base of a right pyramid 2″ high. Show the plan of this solid—

(*a*) When one triangular face is horizontal.

(*b*) When that face is vertical.

N.B. Both these plans must be deduced from the elevation first drawn.

22. A cube of 3·5″ edge has one diagonal vertical. The centre points of the adjacent sides of each face being joined, six squares are obtained, one on each face; these squares are made *the bases of* right pyramids 1·5″ in height. Draw

the plan of the solid and an elevation on any plane not parallel to an edge of the cube.

FIG. 58.

23. A solid is formed of two equal square prisms—side of base $1\frac{1}{2}''$, height $3\frac{1}{2}''$—the axes of which bisect each other at right angles. The elevation is shown, but is not drawn to scale. Draw this elevation the proper size, and deduce the plan, and a second elevation on a new ground line making 50° with X Y.

CHAPTER V.

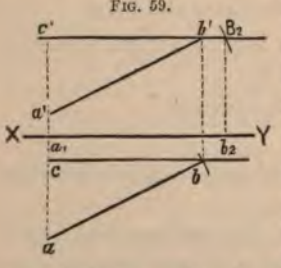
ADDITIONAL PROBLEMS ON LINES.

PROBLEM 23.

Given the true length of a line, or its inclination to one of the co-ordinate planes, and the distances of its extremities from the co-ordinate planes, to determine its projections.

Determine the projections $a\,a'$ of one end A by Problem 1. On $a_1\,a$ mark off a distance $a_1\,c$ equal to the distance of the end B of the line from the vertical plane, and on $a_1\,a'$ mark off $a_1\,c'$ equal to the distance of B from the horizontal plane. Through c and c' draw lines parallel to the ground line. These lines will contain the plan and elevation respectively of the other extremity B of the line. To fix the projections of B we must know the length of $a\,b$ or of $a'\,b'$; one of these can be found by making use of the given true length or inclination thus, with a' as centre and the true

FIG. 59.



length of the line as radius, describe an arc to cut the parallel through c' at B_2. Through B_2 draw $B_2 b_2$ at right angles to X Y. $a_1 b_2$ is the length of the plan of A B, therefore, with a as centre and $a_1 b_2$ as radius, describe an arc to cut the parallel through c at b. $a b$ is the required plan. A perpendicular from b to X Y to meet the parallel through c' determines b'. $a' b'$ is the required elevation.

If the inclination of A B to the horizontal plane is given instead of its true length, the only difference in the construction will be that $a' B_2$ must be drawn so as to make the given angle with X Y.

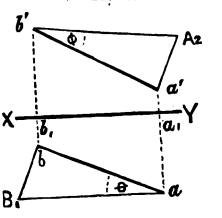
If the inclination to the vertical plane is given, make the above construction on the plan instead of on the elevation.

PROBLEM 24.

Given the projection of a line (A B) on one of the co-ordinate planes, its inclination to that plane and the distance of one end (A) from it; to determine its other projection.

Let $a b$ be the given projection. The distance of a' from X Y is known, being equal to the distance of A from the horizontal plane which is supposed to be given. At a make the angle $b a B_1$ equal to the given inclination and draw $b B_1$ at right angles to $a b$. To the length $b B_1$ add the length $a' a_1$ which will give the distance of B from the horizontal plane, and therefore the distance $b' b_1$.

FIG. 60.



If the elevation $a' b'$ is given instead of the plan, then make the above construction on the elevation instead of on the plan as shown in the figure.

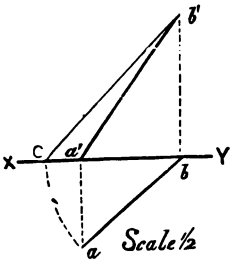
PROBLEM 25.

A line is inclined at $m°$ to one of the co-ordinate planes, and its projection on that plane makes $n°$ with the ground line, its true length being also given; to draw its projections.

Conceive the line to lie on the surface of a cone whose base is in the plane to which the inclination of the line is given, and whose slant side is equal to the given line and is inclined to its base at $m°$. Draw the projections of this cone. One projection of the cone will be a circle, and a radius of this circle inclined at $n°$ to the ground line will be one projection of the line; the other projection of the line will have one extremity at the vertex of the triangle, which is the other projection of the cone, and the other extremity on the base of that triangle.

EXAMPLE. *A line 2″ long is inclined at 45° to the horizontal plane, and its plan makes 40° with the ground line; to draw its plan and elevation.*

FIG. 61.



At a point C in X Y make the angle b' C b equal to 45°, and make C b' 2″ long. Draw b' b perpendicular to X Y. b' C b will be the semi-elevation of the cone previously mentioned. With b as centre and b C as radius, describe the arc C a, which will be part of the plan of the cone. Make the angle C b a equal to 40°. a b is the plan of the line. From a draw a a' perpendicular to X Y: a' b' is the elevation of the line.

PROBLEM 26.

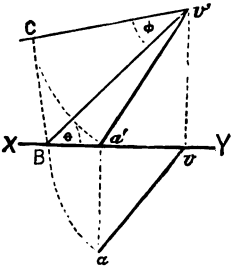
Given the inclinations of a line to the co-ordinate planes, to determine its projections.

Let the line be inclined at θ degrees to the horizontal plane, and ϕ degrees to the vertical plane.

Take any point B in X Y and make the angle v B v' equal

to θ, the inclination of the line to the horizontal plane. Make B v' equal to the length of the line. Draw v' v perpendicular to X Y. Make the angle B v' C equal to ϕ, the inclination of the line to the vertical plane. From B draw B C perpendicular to v' C. With centre v' and radius v' C describe the arc C a', cutting X Y at a'. v' a' is the elevation of the line. With v as centre and v B as radius, describe the arc B a. Draw a' a at right angles to X Y and meeting the arc B a at a. v a is the plan of the line.

FIG. 62.



The line is supposed to lie on the surface of a cone whose base is on the horizontal plane, and whose slant side is equal to and inclined at the same angle as the line. The plan of the line is a radius of the circle which is the plan of the cone, and the elevation has one extremity at the elevation of the vertex of the cone, and the other in the elevation of its base. Since B v' is the true length of the line, and the angle B v' C is equal to its inclination to the vertical plane, v' C, the base of the right angled triangle B v' C is the length of the elevation of the line. Thus the elevation of the line and then its plan can be determined.

Note. The sum of the angles θ and ϕ may vary between 0° and 90°. When $\theta + \phi = 0°$ the projections of the line are parallel to the ground line, and when $\theta + \phi = 90°$ the projections of the line are perpendicular to the ground line.

PROBLEM 27.

Through a given point to draw a line parallel to a given line.

The projections of the required line must pass through the projections of the given point; and since the projections of parallel lines are parallel, the required plan and elevation will be parallel to the given plan and elevation respectively.

PROBLEM 28.

To determine the condition that two straight lines, whose projections are given, may intersect.

If the lines intersect, their point of intersection is a point in each of the lines, therefore the plan of that point must lie on the plan of each of the lines, and therefore the plans of the lines must intersect, and the point of intersection of the plans will be the plan of the intersection of the lines.

By similar reasoning the elevations will meet at a point which is the elevation of the point of intersection of the lines. But the plan and elevation of a point are in the same straight line perpendicular to the ground line.

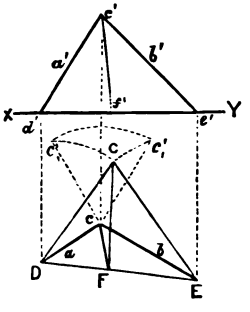
Hence the condition that two lines intersect is that their plans and elevations respectively intersect, and that the points of section are in the same straight line perpendicular to the ground line.

There is one exception to this general rule. When the lines are perpendicular to the ground line, as in fig. 18, they may or may not intersect. In this case an auxiliary plan or elevation must be drawn in order to determine whether they intersect or not.

PROBLEM 29.

Given the projections of two intersecting straight lines, to determine the angle between them.

FIG. 63.



Let A C and B C be the lines whose projections are given. Take the projections of any point, D, in A C, and of any point, E, in B C. Determine by Problem 6 the true form of the triangle D C E. The angle C of this triangle is the angle required.

The construction is simplified in many cases by taking for the points D and E the horizontal or vertical traces of the lines; this is done in fig. 63.

PROBLEM 30.

Given the projections of two straight lines which do not intersect, to determine the angle between them.

Through any point in one of the lines draw a line parallel to the other by Problem 27. Find the angle between these two intersecting lines by Problem 29, which will be the angle required.

PROBLEM 31.

Given the projections of two intersecting straight lines, to draw the projections of the line bisecting the angle between them.

Proceed as in Problem 29 to find the angle between the lines. Bisect the angle D C E (fig. 63) by the line C F, meeting D E at F. Find f', the elevation of F. f' c' is the elevation, and F c is the plan of the line bisecting the angle between A C and B C.

EXERCISES.

1. A line $2\frac{1}{2}''$ long has one extremity $1''$ above the H.P., and $1''$ in front of the V.P. The other extremity is $2''$ above the H.P. and $1\frac{1}{2}''$ in front of the V.P. Draw plan and elevation.

2. Two points, P and Q, are $2''$ apart. P is $1''$ above the H.P. and $\frac{1}{2}''$ in front of the V.P. Q is $1\frac{1}{2}''$ above the H.P. and $1\frac{3}{4}''$ in front of the V.P. Show the projections of the points.

3. The plan of a line is $2''$ long and makes an angle of 35° with X Y. The line itself is inclined at 40° to the H.P., and its ends are in the planes of projection. Draw the projections of the line.

4. The elevation of a line, A B, is $3''$ long, and is inclined at 30° to X Y; one end of the elevation a' being $\cdot 5''$ above X Y. The line itself is inclined to the V.P. at an angle of 30°, and has one end, A, $\cdot 5''$ in front of it. Draw the plan and elevation of the line.

5. A line $2\frac{1}{2}''$ long, inclined at 40° to X Y, and having one

end in X Y, is the elevation of a line which is inclined at 45° to the V.P. Draw the projections of the line.

6. A point 2″ above the H.P. and 1″ in front of the V.P. is the higher extremity of a line inclined at 50°, and stopping at the ground. The direction of the plan makes an angle of 45° with the ground line. Draw its plan and elevation.

7. The elevation of a line makes an angle of 45° with X Y; the true length of the line is 2·75″, and it is inclined at 30° to the H.P. Draw its plan and elevation.

8. The true length of a line is 3″, and it is inclined at 48° to the H.P. Draw the plan and elevation of the line when its ends are ·5″ and 1·5″ in front of the V.P.

9. Draw the plan and elevation of a line (length at pleasure) inclined at 35° to the H.P. and 25° to the V.P.

10. An indefinite line is inclined at 40° to the H.P. and makes an angle of 30° with the V.P., the distance between its traces being 3·5″. Construct its plan and elevation.

11. From a point in the V.P. 2″ from X Y draw a line inclined at 40° and 35° to the horizontal and vertical planes respectively.

FIG. 64.

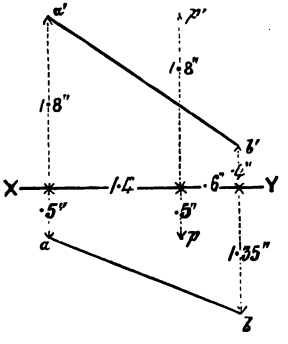
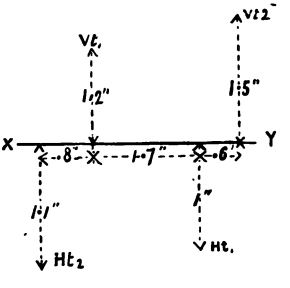


FIG. 65.



12. Show by its traces a line inclined at 40° to the H.P. and 50° to the V.P.

13. Draw the plan and elevation of a line 3″ long when *inclined at* 43° to the H.P. and 33° to the V.P., and show the

plan and elevation of a point in it which is 1″ above the H.P.

14. Represent a line passing through the ground line and making 30° with both planes of projection.

15. Through $p\,p'$ (fig. 64) draw the projections of a line, P Q, parallel and equal to the given line. Join the extremities of the lines and determine the real form of the four-sided figure, A P Q B, obtained.

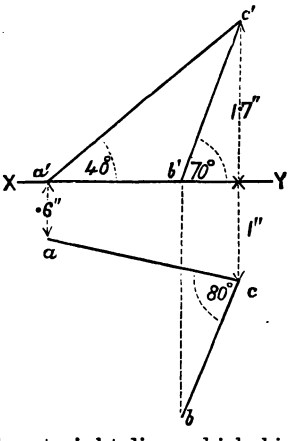
16. Ht_1 Vt_1 (fig. 65) are the horizontal and vertical traces of a line; Ht_2 Vt_2 the corresponding traces of a second line. Do these two lines intersect?

N.B. Fig. 65 is not drawn to scale.

17. Determine the angle between the diagonal of a cube and an adjacent edge; also the true length of the diagonal; Edge of cube, $2\frac{1}{4}''$.

18. What is the real angle between the lines whose projections are given in fig. 66?

FIG. 66.



19. Construct a triangle $o\,a\,b$ ($o\,a = 2\frac{1}{4}''$, $o\,b = 3\frac{3}{4}''$, $a\,b = 4\frac{1}{2}''$). a and b are the horizontal traces of the lines O A, O B, meeting at a point of which o is the plan. The height of O above the H.P. is $1\frac{1}{2}''$. Obtain the real angle contained by the lines.

20. A B is a line parallel to the H.P. A C is a line parallel to the V.P. The angle $b\,a\,c$ between the plans and $b'\,a'\,c'$ between the elevations are each 120°. What is the real angle between the lines?

21. Draw the projections of the straight line which bisects the angle between the two lines given in fig. 66.

CHAPTER VI.

PLANES OTHER THAN THE CO-ORDINATE PLANES.

PLANES other than the co-ordinate planes are represented by the lines in which they meet the latter.

The traces of a plane. The lines in which a plane meets the co-ordinate planes are called the *traces* of that plane, the intersection with the vertical plane being called the *vertical trace*, and that with the horizontal plane the *horizontal trace.*

The method employed for representing planes is illustrated by figures 67 and 68. The perspective figure shows the planes in their true positions, while the other shows them when the co-ordinate planes are made to coincide, as explained in Chapter I.

The traces of a plane either intersect on the ground line or are parallel to it. For since the traces of a plane lie one in each of the co-ordinate planes, if they meet at all the point of intersection must be in the intersection of the co-ordinate planes, *i.e.* on the ground line; and if the traces are parallel to one another they must be parallel to the ground line, for if not they would cut the latter in two points, *i.e.* the plane would meet the ground line in two points, which is impossible unless the plane pass through the ground line, in which case the traces would coincide with that line.

The inclination of one plane to another (called a dihedral angle) is the angle between two straight lines drawn from any point in their common section at right angles to it, one in each plane.

PROBLEM 32.

Given the traces of a plane, to determine its inclination to each of the co-ordinate planes.

In fig. 67 A B is a line in the plane L M N, and perpendicular to M N. *a* B is a line in the horizontal plane, and

also perpendicular to M N. By definition, therefore, the angle A B *a* is the inclination of the plane L M N to the horizontal plane; but this angle is also the inclination of the line A B to the horizontal plane.

Hence, to find the inclination of a plane to the horizontal plane, we require to find the inclination of a line in it which is perpendicular to its horizontal trace. The construction is as follows:—Take any point, *a*, in X Y (fig. 68). Draw *a* B at right angles to M N, and *a* A at right angles to X Y. The line *a* B, fig. 68, is the plan of the line A B, fig. 67, and *a* A

FIG. 67.

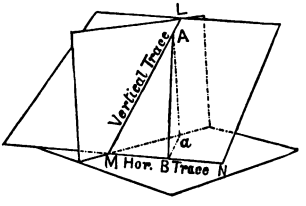
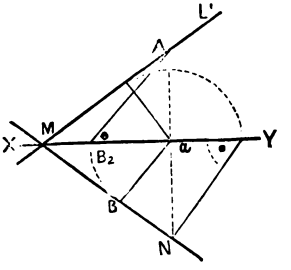


FIG. 68.



is the height of the end, A, above the horizontal plane, the other end, B, being in the horizontal plane. If, therefore, with centre *a* and radius *a* B, the arc B B_2 be described to cut X Y at B_2, the angle A B_2 *a* will be the inclination of the line A B, and also of the plane L M N to the horizontal plane.

The inclination of the plane to the vertical plane is found by a similar construction, which is shown in fig. 68.

The construction just given will apply whatever be the position of the traces. But when the traces are parallel, it is not necessary to make a construction for finding the inclination to the vertical plane after the inclination to the horizontal plane has been found, as the one angle is the complement of the other.

PROBLEM 33.

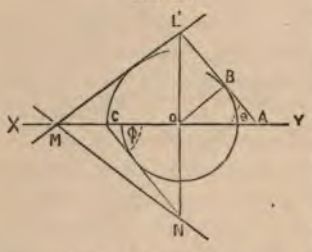
To draw the traces of a plane, having given its inclinations to the co-ordinate planes.

Let the plane be inclined at θ degrees to the horizontal plane and ϕ degrees to the vertical plane.

Note. The sum of the angles θ and ϕ must lie between 90° and 180°.

First method. At any point, A, in X Y make the angle L′ A O equal to θ, the inclination of the plane to the horizontal plane. Draw L′ O N perpendicular to X Y and O B perpendicular to L′ A. With O as centre and O B as radius, describe an arc, and draw N C to touch this arc and meet X Y at an angle equal to ϕ, the inclination of the plane to the vertical plane. With O as centre and O C as radius, describe another arc; the line L′ M drawn through L′ to touch this second arc will be the required vertical trace, and M N the horizontal trace.

FIG. 69.



The theory of this construction is that if two cones be drawn enveloping the same sphere, having their bases in the planes of projection, and their slant sides inclined, at the given angles, the plane which touches these two cones will be inclined to the co-ordinate planes at the same angles as the slant sides of the cones, and therefore at the given angles.

In fig. 69, O is the centre of the sphere, and O B its radius. One cone has its axis in the vertical plane, and the other has its axis in the horizontal plane.

Second method. By Problem 26 draw the projections of a line inclined at $(90 - \theta)$ degrees to the horizontal plane, and $(90 - \phi)$ degrees to the vertical plane. The horizontal and vertical traces of the required plane will be perpendicular to

the plan and elevation of this line respectively, and will of course meet on the ground line or be parallel to it.

PROBLEM 34.

Given one trace of a plane and the inclination of the plane to one of the co-ordinate planes, to determine the other trace.

Case I. When the inclination of the plane is given to the co-ordinate plane containing the given trace L M.

From any point, O, in X Y draw O P at right angles to L M. With O as centre and O P as radius, describe the arc

FIG. 70.

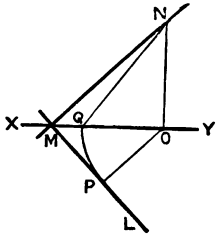
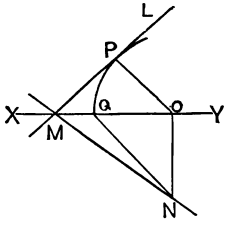


FIG. 71.



P Q, meeting X Y at Q. At Q make the angle O Q N equal to the given inclination of the plane. Draw O N at right angles to X Y, meeting Q N at N. M N is the trace required.

Case II. When the inclination of the plane is given to the co-ordinate plane which does not contain the given trace.

Let M N be the given trace (same figures). At any point, Q, in X Y make the angle O Q N equal to the given inclination of the plane. Draw N O at right angles to X Y. With O as centre and O Q as radius, describe the arc Q P. Through M draw M L to touch this arc. M L is the required trace.

The correctness of the above constructions will become apparent, if after they are finished we proceed in each case as if to find that inclination of the plane which is given in the statement of the problem.

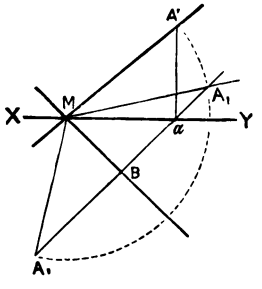
Note. In one figure L M is supposed to be a horizontal trace, and in the other a vertical trace.

PROBLEM 35.

To find the true angle between the traces of a plane.

By the true angle between the traces of a plane is meant the angle between them when the co-ordinate planes are in their natural positions; referring to fig. 67, it is the angle A M B. To find this angle we require the true form of the triangle A M B, which is found thus—From any point, *a*, in X Y (fig. 72) draw *a* B perpendicular to the horizontal trace, and *a* A′ perpendicular to X Y. With M as centre and M A′ as radius, describe an arc cutting *a* B produced at A_1. The angle A_1 M B is the true angle between the traces of the given plane.

Fig. 72.



PROBLEM 36.

Given one trace of a plane and the true angle between the traces, to draw the other trace.

Let L M be the given trace. Make the angle L M P equal to the given angle between the traces. Draw P Q perpendicular to L M, meeting X Y at Q. Draw Q N perpendicular to X Y. With centre M and radius M P describe an arc to cut Q N at N. M N is the trace required.

Fig. 73.

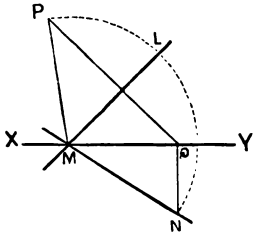
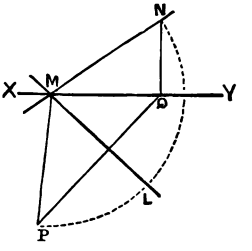


Fig. 74.



Note. In one figure L M is supposed to be a horizontal trace, and in the other a vertical trace.

The correctness of the above construction becomes apparent if, after the required trace is found, we proceed as if to find the true angle between the traces which is given in the statement of the problem.

PROBLEM 37.

To show the intersection of two given planes.

The intersection of two planes is a straight line.

Case I. When the vertical and horizontal traces meet one another respectively.

Let the vertical traces meet at L′ and the horizontal traces at N. It is clear that L′ is a point in the intersection

FIG. 75.

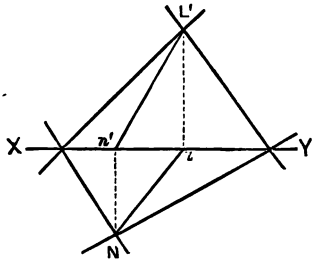
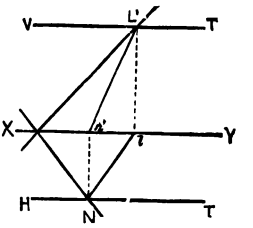


FIG. 76.



of the planes, and being also in the vertical plane its plan, *l*, will be in X Y. It is also evident that N is a point in the intersection of the planes and *n′* its elevation; therefore the line N *l* is the plan and *n′* L′ the elevation of the intersection required.

Case II. When only one pair of traces intersect.

It has been seen that in Case I. one projection of the intersection is got by drawing a perpendicular from the intersection of one pair of traces to X Y, and joining the foot of this perpendicular with the intersection of the other pair of traces. It is clear that in Case II. the first part of this construction can be made as in Case I.; but, as the other pair of

traces are parallel, their point of intersection is at an infinite distance; therefore the line from the foot of the perpendicular on X Y will be parallel to the other traces, *i.e.* one projection

FIG. 77.

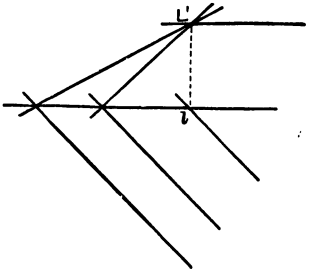
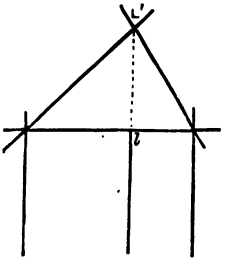


FIG. 78.



of the intersection of the planes is parallel to the parallel traces.

The other projection of the intersection is either a point or a straight line parallel to the ground line.

Case III. When all the traces meet at a point on the ground line or are parallel to it.

FIG. 79.

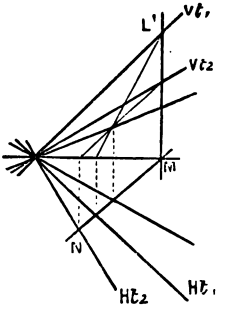
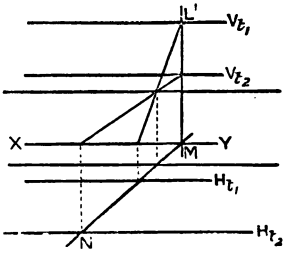


FIG 80.



Let Ht_1 Vt_1 be the traces of one plane, and Ht_2 Vt_2 the traces of another—figs. 79 and 80.

Draw the traces L'M, M N of a third plane (preferably *perpendicular* to one of the co-ordinate planes) to cut each of *the given* planes. Find the lines of intersection of the third

plane with the given planes as in Case I. The point in which these lines meet must be a point in all three planes, and therefore a point in the intersection of the given planes. Lines through the projections of this point, and the intersection of the given traces in the one case (fig. 79), and lines through the projections of this point parallel to the ground line in the other case (fig. 80), will be the projections required.

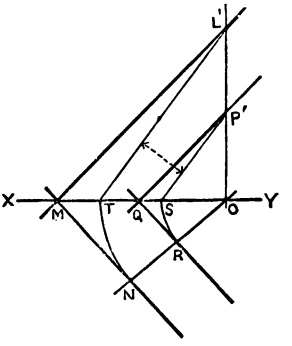
PROBLEM 38.

To determine the distance between two parallel planes.

Note. Parallel planes have parallel traces.

Cut the given planes L'M N, P'Q R by a vertical plane whose horizontal trace, O N, is perpendicular to their horizontal traces. This plane will cut the given planes in two parallel lines, the distance between which will be the distance between the planes.

FIG. 81.



With centre O, and radii O R and O N, describe arcs to cut X Y at S and T. The distance between P'S and L'T is the distance required.

We have here conceived the vertical plane L'O N to rotate about its vertical trace (which is perpendicular to X Y) until it coincides with the vertical plane of projection, carrying with it the lines of its intersection with the given planes.

PROBLEM 39.

To determine the traces of a plane parallel to a given plane, and at a given distance from it.

This problem is the converse of the preceding problem, and will be easily understood.

Let L′M N (fig. 81) be the given plane. Cut this plane by a vertical plane, L′O N, whose horizontal trace, O N, is at right angles to M N. With O as centre, and O N as radius, describe an arc to cut X Y at T. Join L′T, and draw P′S parallel to L′T, and at the given distance from it. P′Q parallel to L′M will be the vertical trace of the plane required, and Q R parallel to M N will be the horizontal trace required.

If the traces of the given plane are parallel to X Y, the horizontal trace, Q R, will be found by drawing the arc S R with centre O, and then a tangent to it parallel to M N.

As two lines can be drawn parallel to the line L′T, and at a given distance from it, there will be two planes which will fulfil the conditions of the problem.

EXERCISES.

1. The vertical trace of a plane makes an angle of 45° with X Y, and the horizontal trace one of 35°. Determine the inclination of the plane to both co-ordinate planes.

2. The traces of three planes are situated as follows :—

(*a*) Both traces are perpendicular to X Y.

(*b*) The horizontal trace makes 48° with X Y, and the vertical trace is at right angles to X Y.

(*c*) The vertical trace is inclined at 20° to X Y, while the horizontal trace is perpendicular to X Y.

What are the inclinations of each of these planes to the planes of projection ?

3. The horizontal trace of a plane is parallel to X Y, and 1·5″ below it. The vertical trace is 2″ above X Y. Find the inclination of the plane to both co-ordinate planes.

4. Show the traces of a plane which is perpendicular to the vertical plane, and inclined at 50° to the horizontal plane.

5. An indefinite plane is to be shown by its traces when it is inclined at 50° and makes an angle of 60° with the vertical plane of projection.

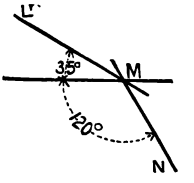
6. A straight line crossing the ground line at an angle of 37° is both horizontal and vertical trace of a plane. Determine its inclination to both planes of projection and the real angle between the traces.

7. Determine the real angle between the traces of the plane in Exercise 1.

8. What is the real angle between the traces of the planes (*a*), (*b*), (*c*) in Exercise 2?

9. Find the inclination of the given plane to both co-ordinate planes, and the real angle between its traces.

FIG. 82.



10. The vertical trace of a plane makes an angle of 40° with X Y, and the plane is inclined at 45° to the horizontal plane. Draw both traces.

11. The horizontal trace of a plane is parallel to the ground line and 2″ below it, and the plane is inclined at 50° to the vertical plane. Find by construction the vertical trace of the plane.

12. A line inclined at 40° to X Y is the horizontal trace of a plane. The real angle between the traces of the plane being 60°, find the vertical trace.

13. The real angle between the traces of a plane is 45°, and the vertical trace makes an angle of 30° with X Y. Represent the plane by its traces.

14. The two traces of a plane form with each other an angle of 50°; on paper they form equal angles with the ground line. At what angle is the plane inclined to each plane of projection?

15. The traces of two planes are as follows:—

H.T. of No. 1 makes 35°, V.T. 70° with the ground line.

H.T. „ 2 „ 50°, V.T. 42° „ „

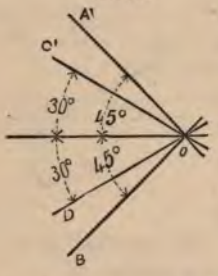
The planes are inclined in opposite directions. Determine the true inclination of their line of intersection.

16. The H.T. of a plane makes 25° with X Y; the V.T. 50°. The horizontal and vertical traces of another plane are each perpendicular to X Y. Find the inclination of their line of intersection.

17. The vertical trace of a plane makes an angle of 50° with X Y, and the horizontal trace one of 40°. The horizontal trace of another plane is parallel to X Y, and 1·5″ below it, the vertical trace being 1·25″ above X Y. Show the line of intersection of the two planes, and find its inclination to both co-ordinate planes.

18. Determine the intersection of the planes A′O B, C′O D and of A′O D, C′O B.

FIG. 83.



19. Draw a straight line making an angle of 50° with X Y, and another bisecting this angle. The former line is both horizontal and vertical trace of a plane, and the latter is both horizontal and vertical trace of another plane. Determine the intersection of the two planes.

20. The vertical trace of a plane is parallel to X Y, and 2″ above it, and the plane is inclined at 60° to the vertical plane; find the point of intersection of this plane with the planes A′O B, C′O D given in Exercise 18.

21. The vertical traces of two planes are each inclined at 45° to X Y and are 1″ apart. The horizontal traces are each inclined at 30° to X Y. What is the distance between the planes?

22. Two parallel planes are parallel to X Y. The H.T. of the first is 1″ and that of the second $1\frac{3}{4}$″ below X Y. The V.T. of the first is $1\frac{1}{2}$″ above X Y. Find the distance between the planes.

23. Draw a line parallel to the H.T. of the plane given in Exercise 9, and 1″ from it; this is the H.T. of a plane parallel to the first. Find the distance between the two planes.

24. The H.T. of a plane makes 45° with X Y, and the plane is inclined at 60°. Draw the traces of a plane parallel to this one and $\frac{1}{2}$″ distant from it.

25. The vertical and horizontal traces of a plane make angles of 48° and 28° respectively with X Y. Show the

traces of two planes parallel to this one and each $\frac{3}{4}''$ distant from it.

26. A line making an angle of 40° with the ground line is both horizontal and vertical trace of a plane. Show the traces of two planes, each parallel to this one and $\frac{1}{2}''$ distant from it.

CHAPTER VII.

PROBLEMS ON THE STRAIGHT LINE AND PLANE.

PROBLEM 40.

Given one projection of a point lying on a given plane, to determine the other projection.

Let the plan, a, of a point A be given (fig. 84). Through a draw $a\,b$ parallel to the horizontal trace of the given plane,

FIG. 84.

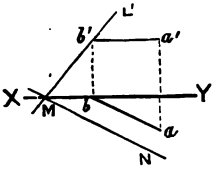
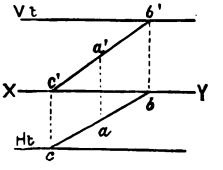


FIG. 85.



meeting X Y at b. Draw $b\,b'$ perpendicular to X Y, meeting the vertical trace of the plane at b'. Through b' draw $b'\,a'$ parallel to X Y, meeting a perpendicular from a to X Y at a'. The latter point is the other projection required.

If the elevation of the point is given its plan may be found by working the above construction backwards.

We have here conceived a horizontal line to pass through the point A and lie in the given plane. The projections of this line must of course pass through the projections of the point A. Also the vertical trace of the line will lie in the

vertical trace of the plane, and the distance of the former from X Y determines the distance of the line, and therefore of the point, from the horizontal plane.

The construction just explained will apply in all cases except the one in which the traces are parallel to the ground line. In this case (fig. 85) through a draw a line, not perpendicular to the ground line, to meet the latter at b and the horizontal trace of the plane at c. Draw $c\,c'$ and $b\,b'$ perpendicular to X Y. Join $b'\,c'$ and draw $a\,a'$ perpendicular to X Y, meeting $b'\,c'$ at a'. a' is the elevation of A.

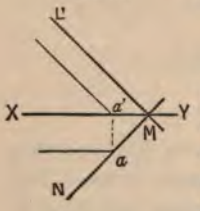
The theory of the construction in this case is that an inclined line is conceived to pass through the given point and lie in the given plane, instead of a horizontal line as in the other cases.

PROBLEM 41.

To draw the projections of a line which is parallel to and at a given distance from one of the co-ordinate planes, and contained by a given plane.

Let the line be parallel to the vertical plane. Its plan will be parallel to the ground line and at a distance from it equal to the distance of the line from the vertical plane.

FIG. 86.



Take a point, a, in the plan of the line (preferably the point where the plan meets the horizontal trace of the plane if it does so within a convenient distance), and find a' by Problem 40. A line through a' parallel to the vertical trace of the plane will be the elevation of the line.

That the elevation of the line is parallel to the vertical trace of the plane is evident, for if the elevation did meet the vertical trace the point of intersection would be a point in the line and a point in the vertical plane; but the line does not meet the vertical plane, being parallel to it, therefore its elevation cannot meet the vertical trace of the plane, *i.e.* it is *parallel to it.*

If the line is horizontal instead of parallel to the vertical plane the construction will be the same as that just explained, if the words vertical and horizontal, and plan and elevation, be interchanged.

PROBLEM 42.

To show the projections of a point which is at given distances from the co-ordinate planes and contained by a given plane.

By the preceding problem draw the projections of a line which shall be contained by the given plane, be parallel to the vertical plane, and at a distance from it equal to the given distance of the point from that plane.

By the same problem draw the projections of a horizontal line contained by the given plane, and at a distance from the horizontal plane equal to the given distance of the point from that plane. The intersection of the plans of these lines will be the plan and the intersection of the elevations will be the elevation of the point required.

PROBLEM 43.

To draw the traces of a plane which shall contain two given intersecting lines or two parallel lines.

If a line be contained by a plane, its traces must lie on the traces of that plane. For the traces of the line are points in the planes of projection, but they are also points in the plane containing the line, therefore the traces of the line must lie on the intersection of the plane with the planes of projection, *i.e.* on the traces of the plane.

Hence the construction is—find the horizontal and vertical traces of the given lines by Problem 5. Join the horizontal traces of the lines to get the horizontal trace, and the vertical traces to get the vertical trace of the plane required.

The traces will either meet at a point on the ground line, or be parallel to it. If the traces of the plane meet, the student will see that after finding one of the traces of the

plane it will only be necessary to find the other trace of one of the lines.

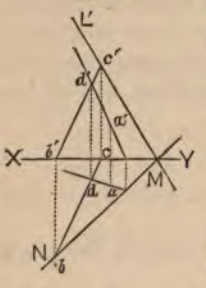
If either of the traces should fall without the paper, draw the projections of a line to meet each of the given lines, but not at their point of intersection. The traces of this new line will also lie on the traces of the plane required.

PROBLEM 44.

To draw the traces of a plane which shall contain a given point and a given line.

Let $a\,a'$ be the given point, and $b\,c$, $b'\,c'$ the given line. Take any point, $d\,d'$, in the given line, and find the traces of the plane containing the lines A D and B C by the preceding problem. These will be the traces of the plane required.

FIG. 87.



PROBLEM 45.

To draw the traces of a plane which shall contain three given points.

Let $a\,a'$, $b\,b'$, and $c\,c'$ be the given points. Join $b\,c$ and $b'\,c'$. Determine by Problem 44 the traces of the plane containing the point $a\,a'$ and the line $b\,c$, $b'\,c'$. These will be the traces required.

PROBLEM 46.

To draw the traces of a plane which shall contain a given line and have a given inclination to one of the co-ordinate planes.

Let $a\,b$, $a'\,b'$ (fig. 88) be the given line, and let the given inclination of the plane be to the horizontal plane.

Draw the projections of a cone having its vertex at the vertical trace of the given line, its base in the horizontal plane, and its slant side inclined at the given inclination (θ).

A line through the horizontal trace of the given line, to

touch the base of the cone, will be the horizontal trace of the plane, and a line through the point where this horizontal trace meets the ground line, and through the vertical trace of the given line, will be the vertical trace of the plane required.

If the horizontal trace of the line fall without the base of the cone, two tangents can be drawn, and there will therefore

FIG. 88.

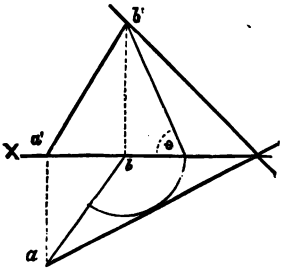
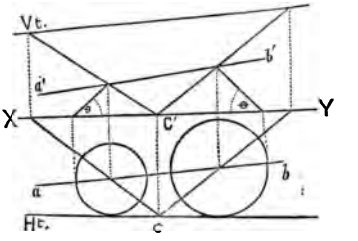


FIG. 89.



be in this case two planes which will fulfil the given conditions. If the horizontal trace of the line fall on the circumference of the base of the cone only one tangent can be drawn, and there will therefore be in this case only one plane which will fulfil the given conditions. If the horizontal trace of the line fall within the base of the cone no tangent can be drawn, which shows that *the problem is impossible when the inclination of the plane is less than the inclination of the line.*

In the construction just explained, we have assumed that the traces of the given line came within the paper; but, as one or both of these traces may fall without the paper, we must show how the construction is modified to meet any case which can occur.

In fig. 89 is shown the solution of the problem when neither of the traces of the given line fall within a convenient distance. In this case the projections of two cones have been drawn, having their vertices in the given line, their bases in the horizontal plane, and their slant sides inclined at the given inclination (θ). The horizontal trace of the plane is a common tangent to the two bases. The vertical trace of the plane *is got by taking* a point c c' in the horizontal trace

just found, and joining this point with two points in the given line. The line joining the vertical traces of these lines is the vertical trace of the plane required.

If the inclination of the plane be given to the vertical instead of to the horizontal plane, the bases of the cones must be placed in the former plane instead of in the latter.

PROBLEM 47.

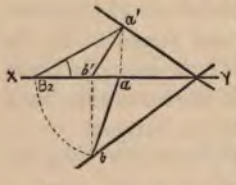
In a given plane to place a line having a given inclination to one of the co-ordinate planes.

The inclination of the line must not exceed the inclination of the plane.

Let the given inclination be to the horizontal plane.

Take a point, B_2, in the ground line and draw $B_2 a'$, making the given angle with X Y, and meeting the vertical trace of the given plane at a'. Draw $a'\,a$ perpendicular to X Y, meeting the latter at a. With a as centre and $a\,B_2$ as radius, describe an arc to cut the horizontal trace of the plane at b. Draw $b\,b'$ perpendicular to X Y, meeting it at b'. $a\,b$ is the plan and $a'\,b'$ the elevation of the line required.

FIG. 90.



The correctness of the construction is apparent if we work the problem backwards in order to find the inclination of the line A B after its projections have been found.

The construction applies whatever be the position of the traces of the given plane.

If the given inclination of the line be to the vertical plane, the construction will be similar to that just explained, and will be easily found out by the student.

PROBLEM 48.

To draw the projections of a line which shall pass through a given point, have a given inclination, and be parallel to a given plane.

In the given plane place a line inclined at the given angle by Problem 47. Through the plan and elevation of the given point draw lines parallel to the plan and elevation respectively of this line. These will be the projections required.

That the line whose projections have thus been found fulfils the given conditions is not difficult to see; for it is parallel to a line which has the given inclination, and must therefore have itself the given inclination. Also it can never meet the given plane, for if it did so it could not be parallel to any line in that plane, therefore it is parallel to the plane.

PROBLEM 49.

To draw the traces of a plane which shall contain a given point, and be parallel to a given plane.

Let Ht Vt be the given plane and $p\,p'$ the given point.

Case I. When the traces of the given plane meet within a convenient distance (fig. 91).

FIG. 91.

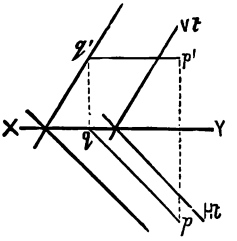
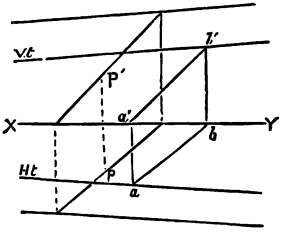


FIG. 92.



Draw $p\,q$ parallel to Ht and $p'\,q'$ parallel to X Y, also $q\,q'$ at right angles to X Y. A line through q' parallel to Vt is the required vertical trace; and a line through the point where this line meets X Y parallel to Ht is the required horizontal trace.

Case II. When the traces of the given plane do not meet within a convenient distance (fig. 92).

In the given plane place a line $a\,b$, $a'\,b'$, inclined at any angle. Draw the projections of a line to pass through P and be parallel to A B. Find the vertical and horizontal traces of the line through P; lines through these parallel to the vertical and horizontal traces respectively of the given plane will be the traces of the required plane.

The construction in each case is evidently correct, for if we examine the figures we see that the plane whose traces have been found contain the given point since it contains a line which passes through that point. Also the planes are parallel because their traces are parallel.

PROBLEM 50.

In a given plane to place a line which shall be parallel to and at a given distance from another given plane not parallel to the first.

Draw the traces of a plane which shall be parallel to the second given plane, and at the given distance from it (Problem 39). The line of intersection of this third plane with the first given plane is the line required.

PROBLEM 51.

To draw the projections of a line which shall pass through a given point and be parallel to two given planes.

The required projections will pass through the projections of the given point, and be parallel to the projections of the line of intersection of the two given planes.

PROBLEM 52.

To draw the traces of a plane which shall contain a given line and be parallel to another given line.

Draw the projections of a line to intersect the first given line and be parallel to the second (Problems 27 and 28). The plane containing these intersecting lines is the plane required. *Its traces* are found by Problem 43.

PROBLEM 53.

To draw the traces of a plane which shall contain a given point and be parallel to two given lines.

Draw the projections of two lines to pass through the given point and be parallel respectively to the two given lines (Problem 27). The plane containing the lines passing through the given point is the plane required. Its traces are determined by Problem 43.

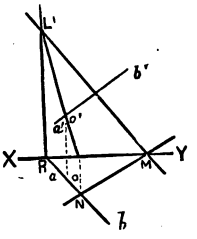
PROBLEM 54.

To show the projections of the intersection of a line and a plane.

Let L'M N be the given plane and $a\,b$, $a'\,b'$ the given line.

Draw the traces, L'R, R N, of a plane (generally perpendicular to one of the planes of projection) to contain the line A B. Find the intersection of this plane with the given plane (Problem 37). The points $o\,o'$ where $a\,b$, $a'\,b'$ meet the projections of the intersection of the planes are the projections required.

FIG. 93.



For the point O is in the line A B, and also in the plane L'M N, since it lies on the intersection of this plane with the plane L'R N, therefore it is the point of intersection of the line and the plane.

PROBLEM 55.

To draw the projections of a line which shall pass through a given point and be perpendicular to a given plane.

If a line is perpendicular to a plane the projections of the line are perpendicular to the traces of the plane, the plan to the horizontal trace and the elevation to the vertical trace.

The construction is therefore—through the plan of the point draw a line at right angles to the horizontal trace of the plane, and through the elevation of the point draw a line at right angles to the vertical trace of the plane; these will be the projections required.

PROBLEM 56.

To draw the traces of a plane which shall contain a given point and be perpendicular to a given line.

Let $p\,p'$ be the given point and $a\,b$, $a'\,b'$ the given line (fig. 94).

Through p draw $p\,q$ perpendicular to $a\,b$, meeting X Y at q. Draw $q\,q'$ perpendicular to X Y, meeting a parallel through p' to X Y at q'. Through q' draw L'M perpendicular to $a'\,b'$ and meeting X Y at M. Through M draw M N perpendicular to $a\,b$. L'M and M N are the vertical and horizontal traces respectively of the plane required.

FIG. 94.

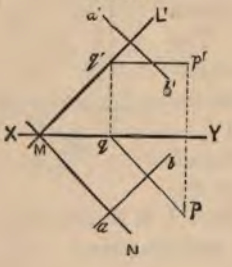
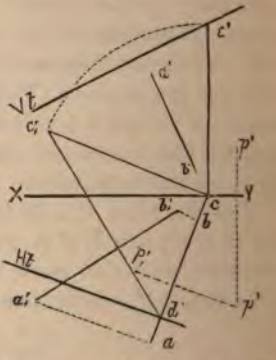


FIG. 95.



In the above construction it is assumed that the line through p perpendicular to $a\,b$ meets X Y within the paper, but as this is not always the case we will give another construction which will apply in any case.

Let $p\,p'$ (fig. 95) be the given point and $a\,b$, $a'\,b'$ the given line.

On $a\,b$ as a ground line make another elevation, p_1', of the point P, and $a_1'\,b_1'$ of the line A B (Problems 15 and 16). Through p_1' draw $c_1'\,d$ perpendicular to $a_1'\,b_1'$, meeting $a\,b$ at d. $c_1'\,d$ is the trace on the vertical plane of which $a\,b$ is the *ground line* of the plane, perpendicular to the line A B and

containing the point P. It is clear that the point d will be a point in the horizontal trace, and that the latter will be determined by drawing through d a line perpendicular to $a\ b$.

Produce $a\ b$ to meet X Y at c, and draw $c\ c_1'$ perpendicular to $a\ b$, meeting $c_1'\ d$ at c_1'. The point c_1' is an auxiliary elevation of a point which is in the required plane, and also in the vertical plane of which X Y is the ground line. If therefore $c\ c'$ be drawn perpendicular to X Y and made equal to $c\ c_1'$, c' will be a point in the vertical trace required which will be perpendicular to $a'\ b'$.

These constructions are easily verified by showing from an examination of the figures that the plane determined fulfils the conditions of the problem.

PROBLEM 57.

To draw the traces of a plane which shall contain a given point and be perpendicular to two given planes.

The plane whose traces are required will be perpendicular to the line of intersection of the two given planes. Hence one construction is—find the intersection of the given planes (Problem 37), and determine the traces of the plane to contain the given point and be perpendicular to this line of intersection (Problem 56).

Another construction is—determine (Problem 55) the projections of two lines which shall pass through the given point, and be perpendicular one to each of the given planes. The plane containing these two intersecting lines is the plane whose traces are required.

PROBLEM 58.

To draw the projections of a line which shall pass through a given point, and meet a given line at right angles.

Determine (Problem 56) the traces of the plane which shall contain the given point and be perpendicular to the given line. Find (Problem 54) the projections of the intersection of this line and plane. The line joining the plan of the given point

with the plan of the intersection of the line and plane will be the plan, and the line joining the elevations of the same points will be the elevation required.

PROBLEM 59.

To draw the traces of a plane which shall contain a given line and be perpendicular to a given plane.

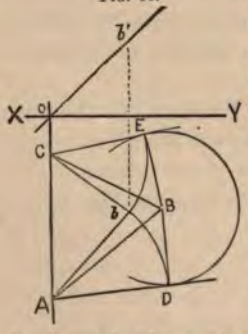
Determine the projections of a line to intersect the given line (Problem 28) and be perpendicular to the given plane (Problem 55). Find (Problem 43) the traces of the plane containing these two intersecting lines. These will be the traces required.

PROBLEM 60.

Given the inclinations of two lines and the angle between them, to draw their projections and the traces of the plane containing them.

Let the angle A B C be the angle between the lines. At any point, A, in A B make the angle B A D equal to the given inclination of A B. Draw B D perpendicular to A D. With centre B and radius B D describe the arc D E, and draw E C to touch this arc, and make an angle with B C equal to the given inclination of B C. Imagine the triangle A B C to rotate about the side A C until the point B is at a distance equal to B D or B E from the horizontal plane, the points A and C being assumed on that plane. In this position the lines A B and B C would be inclined to the horizontal plane at angles equal to B A D and B C E respectively; and their plans would be equal in length to A D and C E respectively. Hence, if with centre A and radius A D the arc D *b* be described, and with centre C and radius C E the arc E *b* be described, meeting the former arc at *b*, *A b and b C* will be the plans of the lines required.

FIG. 96.



A C will be the horizontal trace of the plane containing the lines.

An elevation of the lines on any vertical plane can easily be obtained since the distances of the points A, B, and C from the horizontal plane are known. In the figure an elevation of the lines is shown on a ground line perpendicular to A C. The distance of b' from X Y is equal to B D or B E. O b' will be the vertical trace of the plane containing the lines.

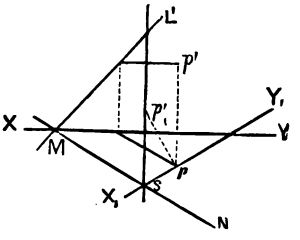
That part of the construction which relates to the drawing of the plan of the given lines is called 'reducing an angle to the horizon.'

PROBLEM 61.

Given the traces of a plane, to determine the trace of this plane on a new vertical plane.

Let L'M, M N be the traces of a plane; it is required to find the trace of this plane on a vertical plane of which X_1 Y_1 is the ground line. Let X_1 Y_1 meet M N at S.

FIG. 97.



Take a point, p, in the new ground line and consider this as the plan of a point lying in the given plane L'M N. Find the elevation p' (Problem 40) of this point on the ground line X Y. Draw pp_1' perpendicular to X_1 Y_1, and make pp_1' equal to the distance of p' from X Y. S p_1' is the vertical trace required. For P is a point in the given plane and also a point in the new vertical plane, therefore p_1' must be a point in the new vertical trace. Also the point S, where the horizontal trace meets X_1 Y_1, is evidently another point in the new vertical trace, therefore the line S p_1' is the trace required.

If the new ground line did not meet the horizontal trace of the plane within a convenient distance, a second point in

the required vertical trace could be found in the same way as the first point p_1'

EXERCISES.

1. Draw a triangle $a\,b\,\mathrm{M}$ (angle $a\,\mathrm{M}\,b = 30°$, angle $a\,b\,\mathrm{M} = 35°$, $b\,\mathrm{M} = 2\frac{3}{4}''$); $a\,\mathrm{M}$ is the horizontal trace of a plane whose vertical trace L/M is parallel to $a\,b$, $b\,\mathrm{M}$ being on the ground line, a point, c, within the triangle $a\,b\,\mathrm{M}$ is in a line through a perpendicular to the ground line and $\frac{1}{2}''$ from it. $a\,b$ is the plan of a line and c is the plan of a point both lying on the plane L/M a. Find their elevations.

2. Three points, A B C, lie on a plane whose vertical and horizontal traces make angles of 35° and 45° respectively with X Y. a' and b' are each 1·25″ above X Y; a' is on the vertical trace of the plane, and b' is 1″ from a'. c' is 2″ above X Y, and at a horizontal distance of 1″ from b'. Show the plans of the points, and determine the true form of the triangle A B C.

3. A triangle A B C lies in a plane L/M N. The vertical trace L/M of the plane makes 39° with X Y, and the horizontal trace M N makes 37° with X Y. a is ·8″ below X Y, and at a horizontal distance of 1·5″ from M. b is ·4″ below X Y, and at a horizontal distance of 3″ from M. c is 1·5″ below X Y and 2·5″ from a. Draw the elevation of the triangle and determine its true form.

4. In the plane given in Exercise 2 place (*a*) a line parallel to the vertical plane, and 1″ in front of it; (*b*) a horizontal line $1\frac{1}{4}''$ above the horizontal plane.

5. The vertical and horizontal traces of a plane make angles of 47° and 38° respectively with X Y. Four points, A B C D, lie in this plane. The points are $\frac{1}{2}''$, $\frac{1}{2}''$, $1\frac{1}{4}''$, and $1\frac{1}{2}''$ respectively above the horizontal plane, and $\frac{1}{4}''$, $1\frac{1}{2}''$, 1″, and $\frac{1}{4}''$ respectively in front of the vertical plane. Draw the projections of the figure A B C D, and show its true form.

6. The horizontal traces of two parallel planes make angles of 30° with X Y. The planes are inclined at 45° to the *horizontal* plane and are $\frac{3}{4}''$ apart. Draw their traces and

show the projections of a line 2″ long, having its extremities one on each plane and 1″ above the horizontal plane.

7. $a\,o\,c$, $b'o'd'$ (fig. 98) are equilateral triangles of $1\frac{1}{2}$″ side. Draw the traces of the plane containing the lines A B, C D.

8. A horizontal line lying in a plane whose traces both make 50° with X Y is ·75″ above the horizontal plane. Draw plan and elevation.

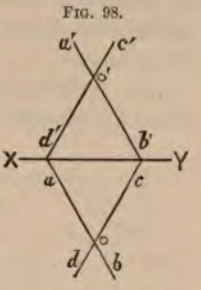


Fig. 98.

9. The elevation $a'b'$ of a line makes 45° with X Y. a' is in X Y and b' is 1·5″ above it. The plan $a\,b$ makes 30° with X Y, and b is in X Y. The elevation of a point P is at b' and its plan is 1·5″ below X Y. Draw the traces of the plane containing the point P and the line A B.

10. Determine the traces of the plane containing the line A B of the preceding exercise and a point Q, whose plan is the same as that of P, but whose elevation is $\frac{3}{4}$″ above X Y.

11. Draw the traces of the plane which shall contain the following three points: A, $\frac{1}{2}$″ above the H.P. and $\frac{1}{4}$″ in front of the V.P.; B, $1\frac{1}{2}$″ above the H.P. and 1″ in front of the V.P.; C, $\frac{3}{4}$″ above the H.P. and $1\frac{1}{2}$″ in front of the V.P. $a\,b = b\,c = 1\frac{1}{4}$″.

12. The plan a of one extremity of a line is 1″ below X Y, and the elevation a' is ·5″ above X Y. The plan b of the other extremity is ·5″ below X Y, and its elevation b' is 1·5″ above X Y. The lines $a\,a'$ and $b\,b'$ are 2·5″ apart. Determine the traces of the plane containing the line A B and inclined at 60°.

13. In a plane whose horizontal and vertical trace make 55° and 32° with the ground line respectively, place a line having the same inclination as the plane, and find the angle between this line and the vertical plane.

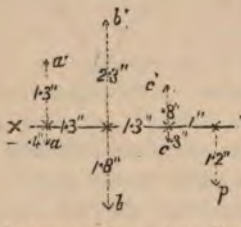
14. In a plane whose traces are each inclined at 40° to X Y, place a line inclined to the vertical plane at the same angle as the plane, and find the inclination of this line to the horizontal plane.

15. Draw the plans of two lines (any length) lying in a

plane inclined at 50° and meeting at a point. The one is inclined at 25°, the other at 35°. Determine the true angle between them.

16. Represent a plane inclined at 50° and two lines lying in this plane meeting one another, the one being horizontal, the other inclined at 30°. Determine the angle contained by these lines.

17. A point whose plan is p lies in the same plane as the three given points $a\,a'$, $b\,b'$, $c\,c'$. Determine its elevation. Solve the problem, if you can, without finding the traces of the plane.

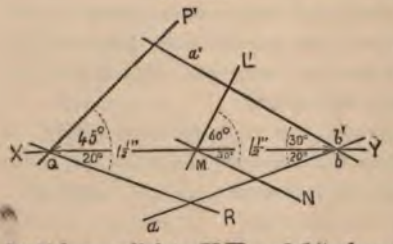


18. Two horizontal lines, A B, A C, contain an angle of 56°. A plane inclined at 30° contains A B, another inclined at 60° contains A C. Draw two lines passing through A, each inclined at 20° and lying one in each plane. Determine the angle between these two lines.

19. The traces, L'M, M N, of a plane make 35° and 50° respectively with X Y. A point, P, 1·5" in front of the vertical plane and ·75" above the horizontal plane, lies in a plane perpendicular to X Y and passing through M. Show the projections of two lines passing through P and parallel to the plane L'M N, one being horizontal and the other inclined at 35°.

20. In the plane P'Q R place a line which shall be parallel to the plane L'M N, and at a distance of $\frac{1}{2}$" from the latter (fig. 100).

FIG. 100.



21. The plans, $a\,b$, $c\,d$, of two lines make angles of 30° with X Y, and meet at a point, c, which is $1\frac{1}{4}$" below X Y. The point o is the horizontal trace of the line C D. $c'\,d'$ makes 45° with X Y. $a'\,b'$ is parallel to X Y and 1" above it. Determine the traces

of the plane which shall contain the line A B and be parallel to the line C D.

22. Find the point of intersection of the line A B (fig. 100) with each of the given planes.

23. The vertical traces of two planes together with the ground line form an equilateral triange of 3″ side. The planes are both inclined at 70° to the horizontal plane. A line is parallel to X Y and 1″ from each of the co-ordinate planes. Find the length of the portion of line intercepted between the two planes.

24. The vertical trace of a plane makes an angle of 55° with X Y and the horizontal trace one of 23°. A point ·3″ below X Y and 1·7″ from the intersection of the traces (measured along X Y) is the plan of a vertical line. Find the intersection of the line and the plane.

25. Find the intersection of the line A B with a plane inclined at 45°, whose horizontal trace is given (fig. 101).

FIG. 101.

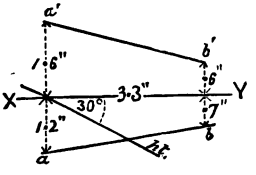


FIG. 102.

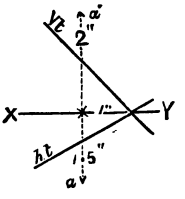
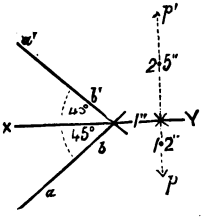


FIG. 103.



26. Find the perpendicular distance of the point P (Exercise 19) from the plane L'M N.

27. Through the given point draw a line perpendicular to the given plane and find its intersection (fig. 102).

28. From the given point $p\,p'$ draw a line to meet the given line $a\,b$, $a'\,b'$ at right angles (fig. 103).

29. Two planes contain a right angle, one of them is inclined to the H.P. at 60°, and their intersection is inclined at 50°. Represent these planes.

30. The horizontal trace of a plane makes 30°, and the vertical trace 45° with the ground line. The plan of a line

lying in this plane makes 24° with the ground line, and its vertical trace is 2·7″ above the H.P. Draw a plane passing through this line at right angles to the given plane.

31. A line, A B, 3″ long, is inclined at 50° to the H.P. and 30° to the V.P.; one end (A) being in the H.P. and the other end (B) in the V.P. From a point, C, in A B, 2″ from A, draw two lines at right angles to A B and to one another, one of the latter being inclined at 30°.

CHAPTER VIII.

SECTIONS OF SOLIDS.

SECTIONS. Many objects of which mechanical drawings have to be made are of such a form that their construction is not completely apparent from outside views only. The interior construction of a house, for instance, cannot be seen from the outside. In order to exhibit the interior of such an object we imagine it to be cut in pieces by planes, and then represent one or more of these pieces separately.

But in representing objects of a comparatively simple nature, the addition of a sectional drawing often adds very much to the illustration of it, although such sectional drawing may not be absolutely necessary to the complete representation of the object.

That surface which is produced when a plane cuts a solid is called in geometry a *section*, and if that part of the solid which is between the cutting plane and the plane of projection is shown on the drawing the latter is called a *sectional plan* or *sectional elevation*. But in the application of these terms to architectural and engineering drawing the word section is often used in the same sense as sectional plan or sectional elevation.

The projection of a section is distinguished in various ways. One way, which we adopt in this work, is by drawing

across it parallel diagonal lines at equal distances apart. These lines are called *section lines.*

If the true form of a section is required, it must be projected on a plane parallel to the plane of the section.

PROBLEM 62.

To draw the projections of the section of a prism by a plane perpendicular to one of the co-ordinate planes.

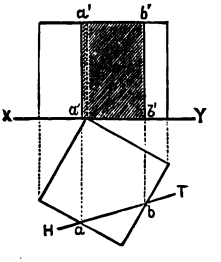
The section will be a plane rectilineal figure, whose angular points are at the points where the edges of the prism meet the cutting plane. Since the plane of section is perpendicular to one of the co-ordinate planes, one projection of the section will be a line coinciding with one of the traces of that plane. Sections of a prism by planes parallel to the ends are similar and equal to the ends.

EXAMPLE 1. *A cube, edge* $1\frac{1}{2}''$, *standing on the H.P. with one face inclined at* 30° *to the V.P., is cut by a vertical plane whose horizontal trace passes through the middle points of two adjacent sides of the base of the cube. To show a sectional elevation.*

First draw the projections of the cube, then the horizontal trace of the cutting plane.

The cutting plane will intersect two of the vertical faces of the cube in vertical lines, of which the points a and b will be the plans and perpendiculars $a'\,a'$, $b'\,b'$ to XY the elevations.

FIG. 104.



The same plane will intersect the horizontal faces of the cube in two horizontal lines, of which the line $a\,b$ will be the plan and $a'b'$, $a'b'$ the elevations.

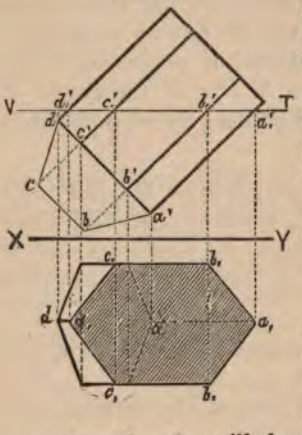
The rectangle $a'\,a'\,b'\,b'$ will be the elevation of the section. The elevation of the part of the solid behind the plane of section completes the sectional elevation.

If the true form of the section is required, it may be shown by an elevation on H.T. as a ground line.

EXAMPLE 2. *A hexagonal prism (base 1″ side, altitude 2½″), having one side parallel to the vertical plane and its axis inclined at 45°, is cut by a horizontal plane into two equal parts. To show the plan of the lower part.*

First construct half of a hexagon, $a'\ b\ c\ d'$, of 1″ side, having a diameter, $a'\ d'$, inclined to X Y at 45° (the complement of the inclination of the axis of the prism). From this the elevation of the prism is got, as shown in the figure.

FIG. 105.



Since the cutting plane divides the solid into two equal parts, its vertical trace will bisect the elevation of the axis of the prism.

The plan will be symmetrical about a line ($a_1\ d$) parallel to X Y.

Draw lines from $a_1'\ b_1'\ c_1'\ d_1'$ perpendicular to X Y, and make the points $b_1\ b_1\ c_1\ c_1$ at distances from the line $a_1\ d$ equal to $b\ b'$ or $c\ c'$.

$a_1\ b_1\ c_1\ d_1\ c_1\ b_1$ will be the plan of the section and will show its true form.

The construction for determining the plan of the part of the solid under the plane of section is apparent from the figure.

PROBLEM 63.

To draw the projections of the section of a pyramid by a plane perpendicular to one of the co-ordinate planes.

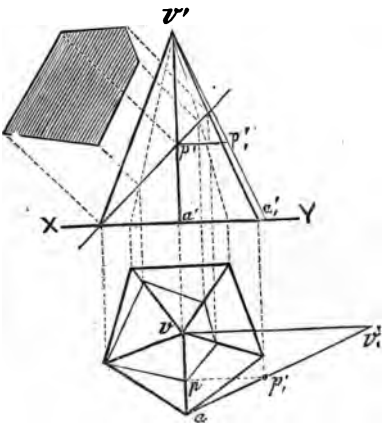
The solution of this problem is similar to that of the preceding one. The section will be a plane rectilineal figure, whose angular points are at the points where the edges of the

solid meet the plane of section. To determine the projections of the section we have, therefore, simply to find the intersections of the edges of the solid with the plane of section, and join them in the proper order. Another way of putting it is as follows: the faces of the pyramid are portions of planes, and the sides of the rectilineal figure forming the section are the lines of intersection of these faces with the plane of section. These lines of intersection may be found from the traces of the faces and of the plane of section by Problem 37.

Sections of a pyramid by planes parallel to its base are similar to the base, but less than it.

EXAMPLE 1. *A pentagonal pyramid (base 1″ side, axis 2″) stands on the H.P. with one side of its base parallel to X Y, and is cut by a plane inclined at 45°, whose horizontal trace is perpendicular to X Y and passes through one of the angular points of the base of the pyramid. To show the plan of the section and its true form.*

FIG. 106.



Commence by drawing the projections of the pyramid and the traces of the plane.

The points where the elevations of the edges of the pyramid meet the vertical trace of the plane are the elevations of the angular points of the section. Perpendiculars from these to X Y to meet the plans of the respective edges will determine the plans of all the angular points of the section excepting one. It will be noticed that the plan $a\,v$ and elevation $a'\,v'$ of one edge are perpendicular to X Y, and that the perpendicular to X Y from p', where $a'v'$ meets the vertical trace of the plane, does not meet $a\,v$ at a point, but coincides with it.

To determine the plan p, draw an elevation of A V on, say, $a\,v$ as a ground line; $a\,v_1'$ is this elevation.

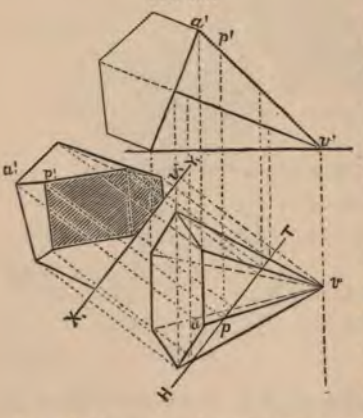
A point, p_1', in $a\,v_1'$, at a distance from $a\,v$ equal to $a'p'$, will be the new elevation of the point P. A perpendicular from p_1' to $a\,v$ determines the plan of the point required.

The plan p may also be determined as follows: make $a'\,a_1'$ equal to $v\,a$, join $a_1'\,v'$, through p', draw $p'\,p_1'$ to meet $a_1'\,v'$ at p_1', then, if $v\,p$ be made equal to $p'\,p_1'$, p will be the plan required.

To show the true form of the section take the vertical trace of the plane of section or a line parallel to it as a ground line, and determine on this a new plan by the rules explained in Chapter IV. This new plan will show the true form of the section since it is a projection of it on a plane parallel to it.

EXAMPLE 2. *A pyramid having a hexagon of* 1″ *side for its base and an altitude of* $2\frac{1}{2}$″, *rests with one triangular face on the H.P. and is cut by a vertical plane whose horizontal trace bisects the plan of the axis of the pyramid at an angle of* 50°. *To show a sectional elevation on a plane parallel to the plane of section.*

FIG. 107.



An elevation and plan of the pyramid when in the given position are first drawn by the rules of Chapter IV. The horizontal trace H.T. of the cutting plane is then drawn, bisecting the plan of the axis at an angle of 50°.

To draw the elevation required a ground line, $X_1\,Y_1$, is taken parallel to H.T.

The points where H.T. meets the plans of the edges of the pyramid are the plans of the angular points of the section.

Consider one of these angular points. $a\,v$, $a'\,v'$ are the projections of one edge of the pyramid. The point p where H.T. meets $a\,v$ is the plan of one angular point of the section. A perpendicular from p to X Y to meet $a'\,v'$ determines p', the elevation of P, on the ground line X Y. The new elevation p_1' of P lies in a line through p perpendicular to X_1Y_1, and is at a height above $X_1\,Y_1$ equal to the height of p' above X Y. In the same way the elevations of the other points of the section are determined. An inspection of the figure will show how the elevation is completed.

It should be noticed that the lines, such as $p_1'\ a_1'$, joining the elevations of the angular points of the section with the elevations of the angular points of the base of the pyramid if produced will meet at a point v_1', which is the elevation of the vertex of the pyramid.

SECTIONS OF THE SPHERE. All plane sections of a sphere are circles. A section by a plane which passes through the centre of the sphere is called a *great circle*, and has a diameter equal to the diameter of the sphere. Any other section is called a *small circle.*

PROBLEM 64.

To determine the projections of the section of a sphere by a plane perpendicular to one of the co-ordinate planes.

One projection of the section will be a line coinciding with one of the traces of the plane of section, and the other will in general be an ellipse.

Let the plane of section be perpendicular to the horizontal plane. Then, the plan of the sphere being a circle, the chord of it which coincides with the horizontal trace of the plane is the plan of the section, and the length of that chord is equal to the diameter of the section.

To draw the elevation is simply to work a problem, which has been already explained in Chapter IV., namely Problem 19.

PROBLEM 65.

Given the projections of a sphere, and one projection of a point on its surface; to determine the other projection of the point.

Take a section of the sphere by a plane parallel to one of the co-ordinate planes, and passing through the point of which one projection is given.

One projection of this section will be a line coinciding with the trace of the plane, and will pass through one projection of the point. The other projection of the section will be a circle which will pass through the other projection of the point. The projections of the point must, of course, be in the same straight line perpendicular to the ground line.

EXAMPLE. *The plan and elevation of a sphere* 2″ *in diameter touch* X Y. *A point, a,* $1\frac{1}{2}$″ *below* X Y, *and* $\frac{5}{8}$″ *from the centre of the plan, is the plan of a point lying on the surface of the sphere; to show the elevation of the point.*

Through a draw a line parallel to X Y, meeting the plan of the sphere at the points b and c. Describe a circle concentric with the elevation, and having a diameter equal to $b\,c$. Through a draw $a\,a'$ perpendicular to X Y, meeting this circle at a'. a' is the elevation required.

As the perpendicular from a will cut the circle in the elevation in two points, the point a will be the plan of two points lying on the surface of the sphere, these points being at equal distances from a horizontal plane passing through the centre of the sphere.

In this example the line $b\,c$ is the horizontal trace of a vertical plane cutting the sphere, and is also the diameter of the section. The circle drawn in the elevation having a diameter equal to $b\,c$ is the elevation of the section.

PROBLEM 66.

Given the projections of a sphere and of a straight line, to determine the projections of the points where the line meets the surface of the sphere.

Let $a\,b$, $a'\,b'$ be the projections of the line, and $o\,o'$ the projections of the centre of the sphere.*

Represent a plane perpendicular to one of the co-ordinate planes, and containing the given line. One trace of this plane will coincide with the plan or elevation of the given line, according as the plane selected is perpendicular to the horizontal or vertical planes of projection.

Suppose the plane selected to be vertical.

The horizontal trace of the assumed plane will coincide with $a\,b$, the plan of the line.

Take a ground line, $X_1\,Y_1$, parallel to $a\,b$, and determine on this new ground line the elevation $a_1'\,b_1'$ of the line A B. If the plan $a\,b$ meet the circle, which is the plan of the sphere, at the points a and b, $a\,b$ will be the diameter of the circle, which is the section of the sphere by the assumed vertical plane. Determine the elevation of this circle on $X_1\,Y_1$. The elevation will be a circle whose diameter is equal to $a\,b$, and whose centre o_1' is at a distance from $X_1\,Y_1$ equal to the distance of o' above X Y.

Let $a_1'\,b_1'$ meet this circle at the points p_1' and q_1'. Through p_1' and q_1' draw perpendiculars to $X_1\,Y_1$ to meet $a\,b$ at p and q. Through p and q draw perpendiculars to X Y, to meet $a'\,b'$ at p' and q'. $p\,p'$ and $q\,q'$ are the points required.

The theory of this construction is simple. The assumed plane contains the given line, and cuts the surface of the sphere in a circle. This circle and line being in the same plane, if the projections of the line meet the projections of the circle this shows that the line meets the circle; but as the circum-

* No figure is given to illustrate this problem, but the student can easily make one for himself.

ference of the circle lies on the surface of the sphere, being a section of it by a plane, the points where the projections of the line meet the projections of the circle must be the projections of the points where the line meets the surface of the sphere.

SECTIONS OF THE CYLINDER. All sections of a right circular cylinder by planes parallel to its axis are rectangles, two sides of which lie on the curved surface of the cylinder, and the other two on the ends.

All sections by planes perpendicular to the axis are equal circles. All other sections by planes which do not cut an end of the cylinder are ellipses, the minor axes of which are equal to the diameter of the cylinder; but if the plane of section also cuts one or both ends of the cylinder the section will be a portion of an ellipse.

PROBLEM 67.

To determine the projections of the section of a cylinder by a plane perpendicular to one of the co-ordinate planes.

In defining a right circular cylinder, it was pointed out that it might be considered as 'a solid described by the revolution of a rectangle about one of its sides, which remains fixed.' If, therefore, a number of lines be drawn to represent a number of different positions of that side of the rectangle which is parallel to the fixed side, these lines being on the surface of the cylinder, their intersection with the cutting plane will determine points in the section required.

By taking a sufficient number of these lines we may determine as many points in the section as we choose. Then by drawing a 'fair curve' through these points the complete section is determined.

EXAMPLE. *A cylinder 2″ in diameter, having its axis inclined at 50°, is cut by a horizontal plane. To draw the ellipse which is the plan of the section.*

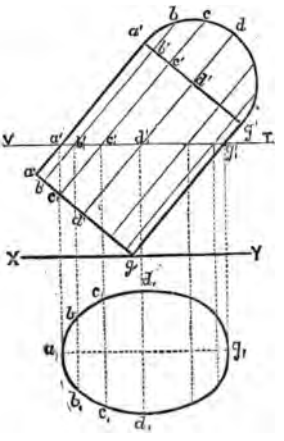
Draw an elevation $a'a'g'g'$ of the cylinder on a plan

parallel to its axis. On $a'g'$ describe a semicircle. This will be a half end view of the cylinder. Draw V.T., the vertical trace of the cutting plane. Draw the lines $b'b'$, $c'c'$, $d'd'$, &c., parallel to the elevation of the axis of the cylinder, and produce them to meet the semicircle at the points b, c, d, &c.

These lines and points are the projections of lines which lie on the curved surface of the cylinder; and these latter lines meet the plane of section at points whose elevations are b_1', c_1', d_1', &c. These points are therefore elevations of points in the curve of section required.

Conceive a plane parallel to the vertical plane of projection, and containing the axis of the cylinder. Draw the line $a_1 g_1$ parallel to X Y to represent a portion of the horizontal trace of this plane.

FIG. 108.



This central plane will divide the cylinder into two equal portions exactly alike. One half of the curve of section formed by the horizontal cutting plane will be in front of this plane, and the other half behind it.

Consider the point B_1 of the section. The plan of this point will lie in the perpendicular through b_1' to X Y. Also the distance of B_1 from the central plane just mentioned will be equal to $b\,b'$, the distance of b from the diameter of the semicircle, therefore the plan b_1 will be at this distance from the line $a_1 g_1$.

It is evident that there will be two points B_1, one in front of the central plane and the other at an equal distance behind it.

In like manner c_1, d_1, &c., the plans of the other points, are obtained.

PROBLEM 68.

Given the projections of a cylinder and one projection of a point on its surface, to determine the other projection of the point.

Take a section of the cylinder by a plane parallel to its axis, perpendicular to one of the co-ordinate planes, and passing through the point of which one projection is given.

The section of the cylinder by this plane will be a rectangle, one projection of which will be a straight line coinciding with one trace of the assumed plane, and will pass through one projection of the point. The other projection of the section will be a parallelogram, one side of which will pass through the other projection of the point. Then, again, the plan and elevation of the point lie in the same straight line perpendicular to the ground line. Thus, if one projection of the point is given, the other can be found.

EXAMPLE. *A cylinder* $1\frac{1}{2}''$ *in diameter has its axis parallel to the ground line and* $\frac{3}{4}''$ *from each plane of projection. A point* $1''$ *from X Y is the plan of a point lying on the surface of this cylinder: to show its elevation.*

Draw $X_1 Y_1$ perpendicular to X Y, and on this new ground line draw an elevation of the cylinder. This elevation will be a circle $1\frac{1}{2}''$ in diameter and touching $X_1 Y_1$. Through p draw a line perpendicular to $X_1 Y_1$, meeting the circle at the points p_1', p_2'. On the first elevation of the cylinder draw two lines parallel to X Y, and at distances from it equal to the distances of p_1', p_2' from $X_1 Y_1$.

Through p draw a perpendicular to X Y to meet these parallel lines. The points thus determined will be the elevations of points of which p is the plan.

SECTIONS OF THE CONE. In what follows we neglect the base of the cone and consider its curved surface only.

The section of a cone by a plane which contains its vertex *is two* intersecting straight lines. If, in addition to contain-

ing the vertex of the cone, the plane of section contain its axis, the section is called a *principal section.*

The section of a cone by a plane perpendicular to its axis is a circle.

If the plane of section cut both lines of a principal section on the same side of the vertex the section is an ellipse.

If the plane of section be parallel to one of the lines of a principal section, and perpendicular to the plane of that principal section, the section is a parabola.

Any other section of the cone is a hyperbola.

The circle may be considered as a particular case of the ellipse, and two straight lines as a particular case of the hyperbola. A single straight line is also a particular case of the hyperbola, or of the parabola. In this case the plane of section touches the cone.

PROBLEM 69.

A cone having its axis vertical is cut by a plane perpendicular to the vertical plane of projection: to determine the plan of the section and its true form.

The solution of this problem is similar to that of Problem 67. In this case we draw lines $v\,a\;v'\,a'$, $v\,b\;v'\,b'$, &c., to represent a number of different positions of the hypotenuse of the right angled triangle, which describes the curved surface of the cone.

The points where these lines meet the plane of section are points in the section required.

Let $v'\,a'$ meet the vertical trace of the plane of section at the point a_1'. Through a_1' draw a perpendicular to X Y to meet $v\,a$ at the point a_1. a_1 is the plan of a point in the curve of section.

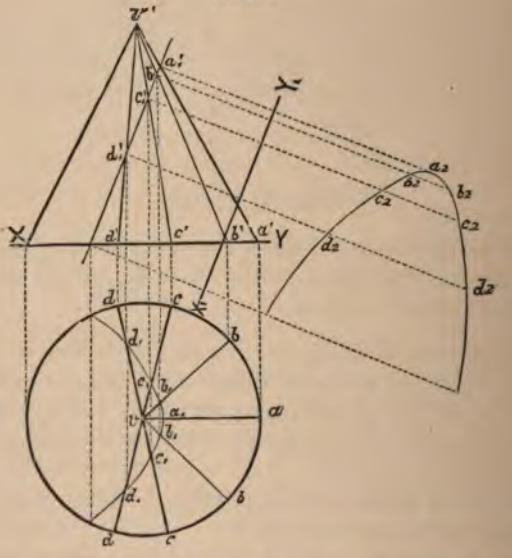
In like manner the points b_1, c_1, &c., are determined, and a fair curve drawn through these completes the plan of the section.

To determine the true form of the section, take the vertical trace of the plane of section or a line parallel to it as a new

ground line. Through the points a_1', b_1', c_1', &c., draw perpendiculars to this new ground line, and make the points a_2, b_2, c_2, &c., at distances from the new ground line equal to the distances of the points a_1, b_1, c_1, &c., from X Y respectively. A fair curve joining these points thus determined will be the true form of the curve of section, which is, in the particular case shown in the figure, a hyperbola.

There are some cases, and fig. 109 shows one of these, where the lines on the surface of the cone meet the plane of

FIG. 109.



section at so acute an angle that it is difficult to determine exactly their points of intersection. In such cases it is better to proceed as follows.

All points in the moving line which describes the curved surface of the cone describe circles whose planes are perpendicular to the axis of the cone. Draw the projections of the circles described by a number of these points. The intersection of these circles with the plane of section determines

points in the section required. When the axis of the cone is vertical the elevations of these circles will be straight lines parallel to the ground line, and their plans will be circles.

This method should also be applied for determining those points which lie on those lines whose projections are perpendicular, or nearly perpendicular, to the ground line when the first method is employed.

PROBLEM 70.

Given the projections of a cone, and one projection of a point on its surface, to determine the other projection of the point.

Let V be the vertex of the cone, and P the point on its surface. Suppose p the plan of the point P to be given. Through v draw a line, $v\,p\,a$, meeting the plan of the base of the cone at the point a. Through a draw $a\,a'$ perpendicular to the ground line, and meeting the elevation of the base of the cone at a'. Join $v'\,a'$, and through p draw $p\,p'$ perpendicular to the ground line, and meeting $v'\,a'$ at p'. p' is the projection required.

If p' is given and p has to be found, the construction would be that just given worked backwards.

If the projections $v\,a$ and $v'\,a'$ are perpendicular to the ground line, the perpendicular $p\,p'$ will not meet either of these at a point, but will coincide with them; in this case it will be necessary to take another elevation to find the distance of p or p' from the ground line.

In this construction $v\,a$, $v'a'$ is a line lying on the surface of the cone, and, as one projection of this line is made to pass through one projection of the point on the surface of the cone, the other projection of the line must pass through the other projection of the point.

We may also consider the line $v\,a$, $v'a'$ to be one of the lines in which the surface of the cone is cut by a plane. This was the method adopted in solving the corresponding problem for the cylinder.

EXERCISES.

1. A pentagonal prism (pentagon $1\frac{1}{4}''$ side, axis $2\frac{3}{4}''$) rests with one side on the ground. Draw a sectional elevation on a vertical plane, bisecting the axis of the prism at an angle of 55°.

2. Draw an equilateral triangle of $3\frac{1}{4}''$ side and another within it having its sides $\frac{1}{2}''$ distant from the sides of first. Draw a line bisecting one side of the larger triangle at an angle of 85°. The triangles form the plan of a hollow prism standing on the H.P., and whose height is $\frac{3}{4}''$. The line is the horizontal trace of a vertical plane cutting the prism into two portions. Draw an elevation of the larger portion looking at right angles to the section.

3. A right rectangular prism—base $1\frac{1}{2}'' \times \frac{3}{4}''$, length $2\frac{1}{2}''$—has the longer sides of its base horizontal and the shorter sides inclined at 45°. The solid is divided into two portions by a vertical plane, which bisects the lowest horizontal edge at an angle of 40°. Show an elevation of the larger portion on a plane parallel to the cutting plane.

4. A right prism whose ends are hexagons of $1{\cdot}25''$ side and whose axis is $4''$ long, lies with one side on the H.P. It is cut into two equal parts by a vertical plane which makes an angle of 40° with its axis. Show the plan of one of these halves when standing on its section end.

5. The base of a hollow prism is a regular hexagon of $1\frac{1}{2}''$ side, with another concentric hexagon of $1''$ side, the sides of the latter being respectively parallel to those of the former. This prism, which is $3''$ long, is halved by a plane which is perpendicular to one side of the prism and inclined at 50° to its ends. Draw the plan of one of these halves when standing on its section end.

6. A square pyramid—base $2''$ side, axis $3''$—stands on the H.P., and is cut by a horizontal plane which bisects the axis. Draw the plan of the frustrum.

7. Shew the plan of the lower portion of the pyramid in

the preceding exercise when the cutting plane bisects the axis at an angle of 45°.

8. A square pyramid (side of square 2·5″, altitude 3·5″) stands on the H.P., and is cut by a vertical plane passing through one point half way down one edge and another point one quarter of the way down an adjacent edge. Show the true form of the section.

9. A hexagon, A B C D E F, of $1\frac{1}{4}''$ side, is the base of a right pyramid, 3″ high, which stands on the H.P. A line cutting C D ·3″ from D and A F ·2″ from F is the horizontal trace of a vertical plane cutting the solid. Show the true form of the section made.

10. A pyramid, having for its base a square of 2·5″ side and its axis 3·25″ long, rests with one triangular face on the ground. Draw its plan and a sectional elevation on a vertical plane, represented by a line bisecting the plan of the axis and making an angle of 60° with it.

11. A heptagonal right pyramid of 1″ edge of base and 2″ high is lying with one face on the H.P., assuming it to be cut by a vertical plane passing through the centre of the heptagonal base, and making an angle of 45° with the plan of its axis. Draw the section and elevation.

12. A hexagonal prism (side of base $1\frac{1}{2}''$, height $1\frac{1}{4}''$) having its base on the H.P. and one side inclined at 10° to the V.P., is surmounted by a tetrahedron having the corners of its base at three of the angular points of the prism, one corner coinciding with that corner of the prism which is nearest to the V.P. Show the plan of the section made by a plane which bisects the axis of the tetrahedron is perpendicular to the V.P., and inclined at 50° to the H.P.

13. A sphere $2\frac{1}{2}''$ in diameter is cut by a plane which is at a distance of $\frac{1}{2}''$ from its centre and inclined at 45°. Show the plan of the section and its true form.

14. A sphere 3″ in diameter has its centre in the ground line. Show the projections of a point on its surface which is 1′ above the H.P., and $\frac{1}{2}''$ in front of the V.P.

15. Draw the projections of the sphere and point as in the preceding exercise. A straight line enters the sphere at this

point, and has its plan and elevation inclined at 60° and 30° respectively to the ground line. Determine the projections of the point where this line leaves the sphere.

16. A cylinder 2″ in diameter and 3″ long has its axis inclined at 40°. Show the true form of the section made: (*a*) by a horizontal plane bisecting the axis of the cylinder; (*b*) by a vertical plane whose horizontal trace bisects the plan of the axis of the cylinder at right angles.

17. A cylinder $2\frac{1}{2}$″ in diameter and $2\frac{1}{2}$″ long has a hexagonal hole through it (side of hexagon $\frac{3}{4}$″), the axis of the hole coincides with the axis of the cylinder, which is horizontal, and it has one side vertical. This solid is cut by a vertical plane, whose horizontal trace coincides with one diagonal of the square which is the plan of the cylinder. Show a sectional elevation looking at right angles to the section.

18. A cylinder $1\frac{1}{2}$″ in diameter rests on the ground, with its axis horizontal and inclined at 30° to the V.P. Two points, each 1″ above the ground and $1\frac{1}{4}$″ in front of the V.P., lie on the surface of this cylinder. Show the projections of the points.

N.B.—The cylinder may be shown of any convenient length, and the ends may be shown by broken lines.

19. A cone (base 3″ diameter, axis $3\frac{1}{2}$″) is cut by a plane bisecting its axis at an angle of 50°. Show the true form of the section.

20. A circle 1·25″ radius is the plan of a right cone 3·5″ high, standing on the horizontal plane. Show the plan and the true form of the section of this cone by a plane whose horizontal trace is 2″ distant from the centre of the plan, and whose inclination is 40°.

21. A right cone—radius of base 1·5″, height 3·5″—stands on the horizontal plane, and is cut by a vertical plane $\frac{1}{2}$″ from the axis. Determine the true form of the section. Name the curve obtained.

22. Determine the plan and true form of a parabolic section of a cone whose base is horizontal by a plane which cuts its axis at a point 1″ from the vertex. Base of cone $2\frac{1}{2}$″ in diameter, axis 3″ long.

23. A triangle $v'\,a'\,b'$ ($v'\,a' = v'\,b' = 3''$, $a'\,b' = 2\frac{1}{2}''$) is the elevation of a right circular cone standing on the ground. Two points, P and Q, lie on the surface of this cone. p' is $1''$ from a' and $2''$ from b'. q' is $2''$ from a' and $2''$ from b'. Draw the projections of the cone, and show the plans of the points P and Q.

CHAPTER IX.

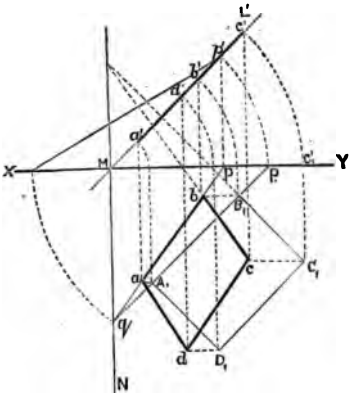
PROJECTION OF PLANE FIGURES.

PROBLEM 71.

Given the inclination of the plane of any plane figure and of any line in that figure, to draw its projections.

First of all determine the traces L'M, MN of the plane containing the figure. It will be found that the working of the problem is much simplified by first assuming this plane perpendicular to the vertical plane, so that its horizontal trace is perpendicular to the ground line, and its vertical trace inclined to the ground line at an angle equal to the inclination of the plane. In this position the elevation of the figure will be a straight line coinciding with the vertical trace of the plane. Afterwards any other elevation may be obtained by Problem 19.

FIG. 110.



Next place in this plane L'M N, by Problem 47, a line, P Q, having the inclination of the line of the figure given in the problem.

Now imagine this inclined plane, with the line P Q upon it, to rotate about its horizontal trace M N until it comes into the horizontal plane.

The point Q being in M N will remain fixed during this rotation, while the point P will describe, an arc of a circle in the vertical plane with M as centre. Hence if with M as centre the arc $p' P_1$ be described, meeting X Y at P_1, $P_1 q$ will be the position of the line P Q when the latter is brought into the horizontal plane.

Mark off on $P_1 q$ a length, $A_1 B_1$, equal to the length of that line of the figure whose inclination is given, and on $A_1 B_1$ construct the given figure whatever it may be.

Note. The line A B is not necessarily a side of the figure, but may be any line whatever in the same plane with it occupying a known position in relation to the figure.

Now imagine the figure thus drawn on the horizontal plane to rotate about M N. The elevation of this figure in any position as it rotates will be a portion of a line passing through M. All points in the figure will describe arcs of circles whose plans will be straight lines perpendicular to M N, and whose elevations will be arcs of circles having their centres at M, and having radii equal to the distances of the points from M N.

Thus the point C will describe an arc whose plan is the straight line $C_1 c$ perpendicular to M N, and whose elevation is the arc $c_1' c'$.

Let the rotation continue until the plane of the figure has the required inclination, *i.e.* until the elevation coincides with L'M. Perpendiculars from the elevations of the angular points of the figure to X Y to meet the plans of the arcs described by them during the rotation determines the plans of these points.

Thus a perpendicular from c' to X Y to meet the line through C_1 perpendicular to M N determines c, the plan of one angular point of the figure.

As a verification of the construction the student should *notice that* if any side, say $B_1 C_1$, of the figure constructed on *the horizontal* plane be produced to meet M N, it will do so at

the point where the plan $b\,c$ of the same line in its inclined position meets it.

It must be borne in mind that no line of the figure can have a greater inclination than that of its plane.

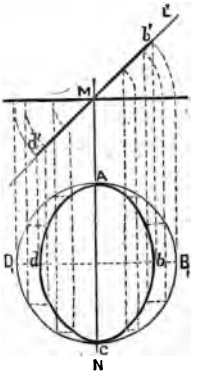
PROBLEM 72.

Given the inclination of the plane of a circle, to draw its projections.

Draw L'M, M N, the traces of the plane containing the circle, as in the preceding problem.

On the horizontal plane draw the circle $A\,B_1\,C\,D_1$, having a diameter equal to that of the given circle.

Fig. 111.



Through any convenient number of points in this circle draw perpendiculars to X Y. With M as centre, and the distances of the feet of these perpendiculars from M as radii, describe arcs of circles to cut L'M. From the points where these arcs cut L'M draw perpendiculars to X Y, to meet perpendiculars to M N from the corresponding points in the circle $A\,B_1\,C\,D_1$. This determines points in the plan of the circle, and by drawing a fair curve through these the plan will be complete. The plan will be an ellipse, whose major axis is the plan of A C, that diameter of the circle which is parallel to M N and therefore horizontal. (In fig. 111, A C has been made to coincide with M N.) The minor axis of the ellipse is the plan of B D, that diameter of the circle which is perpendicular to M N.

We may therefore determine the axes of the ellipse, and then draw it by any of the rules for drawing ellipses.

The line $d'\,b'$ is the elevation of the circle.

From this plan and elevation any other elevation may be obtained by *Problem* 19.

PROBLEM 73.

Given the inclinations of any two intersecting straight lines in a plane figure, to draw the projections of the figure.

Determine by Problem 60 the traces of the plane containing the lines whose inclinations are given, taking the ground line perpendicular to the horizontal trace.

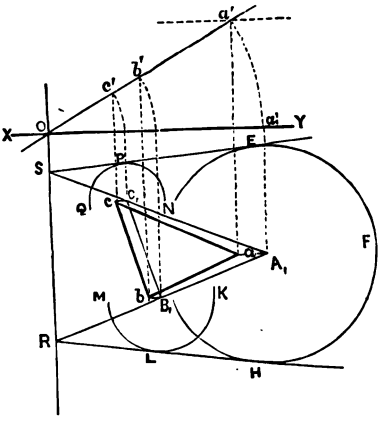
Rotate this plane with the lines upon it about its horizontal trace, as in Problem 71, until it comes into the horizontal plane.

On the lines thus brought into the horizontal plane construct the given figure, and proceed exactly as in Problem 71 to determine the plan.

PROBLEM 74.

Given the heights of three points in a plane figure, to determine its projections.

FIG. 112.



Note. The difference between the heights of any two points must not exceed the distance between the points.

The three points if joined will form a triangle, and we

have first to determine the traces of the plane containing this triangle.

Let A, B, and C be the points whose heights are given.

Construct on the horizontal plane a triangle, $A_1 B_1 C_1$, equal to the triangle A B C.

With centre A_1 and radius equal to the height of the point A, describe the circle E F H. With centre B_1 and radius equal to the height of the point B, describe the circle K L M. With centre C_1 and radius equal to the height of the point C, describe the circle N P Q.

Draw H L to touch the circles E F H and K L M, and produce it to meet $A_1 B_1$ produced at R. Also draw E P to touch the circles E F H and N P Q, and produce it to meet $A_1 C_1$ produced at S. R S is the horizontal trace of the plane required.

Make X Y perpendicular to R S, meeting the latter at O. Draw $A_1 a_1'$ perpendicular to X Y. With centre O and radius $O a_1'$, describe an arc, $a_1' a'$, to meet at a' a parallel to X Y, which is at a distance from the latter equal to the height of the point A. $O a'$ is the vertical trace of the plane required.

The theory of this construction is similar to that of Problem 60, which the student should refer to again.

It is evident that the angle $A_1 R H$ is the inclination of A B, and that the angle $A_1 S E$ is the inclination of A C.

The plan $a b c$ of the triangle A B C is now determined as shown in the figure, which is just a repetition of the construction performed in the preceding problems of this chapter.

If A B C is not the complete figure given in the problem, a figure equal to it must be built up on the triangle $A_1 B_1 C_1$. Then the plan of the whole figure is obtained in the same way as the plan of the part A B C.

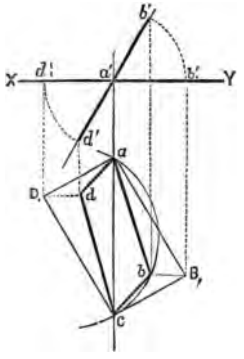
PROBLEM 75.

A plane figure revolves about a fixed horizontal line in it till the plan of an angle opposite to that line is equal to a given angle: to determine the projections of the figure.

Let A B C D denote the given figure, A C the line about which it is to revolve, and B the angle whose plan is to be equal to a given angle.

Construct the figure $a\,B_1\,c\,D_1$ equal to the figure A B C D. On $a\,c$ describe by Euclid III. and 33 a segment of a circle to contain an angle equal to that into which the angle B is to be projected.

FIG. 113.



As the figure revolves about A C, the point B will describe an arc of a circle whose plan is a line through B_1 perpendicular to $a\,c$. Let this perpendicular meet the segment of a circle which was described on $a\,c$ at the point b. $a\,b\,c$ is the plan of the angle B when the figure has revolved as stated in the problem.

Draw X Y perpendicular to $a\,c$, meeting the latter produced at a'. Draw $B_1\,b_1'$ perpendicular to X Y. With centre a' and radius $a'\,b_1'$ describe the arc $b_1'\,b'$, and through b draw a perpendicular to X Y to meet this arc at b'. $a'\,b'$ is the vertical trace of the plane containing the figure A B C D when the latter occupies the position stated in the problem.

From D_1 draw $D_1\,d_1'$ perpendicular to X Y. With centre a' and radius $a'\,d_1'$ describe an arc to cut $a'\,b'$ at d'. Draw $d'\,d$ perpendicular to X Y, to meet a line through D_1 perpendicular to $a\,c$ at d. $a\,b\,c\,d$ is the plan and $d'\,b'$ an elevation of the figure A B C D, as required.

EXERCISES.

1. Draw the plan of an equilateral triangle of 2·5″ side, its plane to be inclined at 48° and one side at 32°.

2. Draw the plan of an isosceles triangle, whose base is 2·5″ and sides 3″ when its plane is inclined at 50° and its base at 35°.

3. The sides of a triangle are 3″, 2·75″, and 2·5″ long respectively. Draw its plan when the plane of the figure is inclined at 55° and the longest side at 30°.

4. Draw the plan of a rectangle 2″ × 3″ when its plane is inclined at 50° and one diagonal at 35°.

5. A rectangle, sides 2″ and 3″, has its plane inclined at 60° to the H.P., the short sides being horizontal. What is the inclination of its diagonals?

6. Construct the plan and two elevations of a square of 2·5″ side when its plane is inclined at 50° and one edge is inclined at 30° (one of the elevations to be on a plane parallel to one diagonal).

7. Draw the plan of a rectangle (sides 3″ and 2″) when its plane is inclined at 50° and one diagonal is horizontal. Also show an elevation on a plane parallel to the other diagonal.

8. A pentagon, A B C D E, side 1·75″, revolves upon the line joining A with the centre of the opposite side till its plane becomes inclined at 50°. Draw its plan.

9. The plane of an isosceles triangle (base 2″, sides 3·5″) is vertical and inclined at 40° to the vertical plane. One side is inclined at 70°. Draw the elevation of the triangle.

10. Both the traces of a plane make angles of 45° with X Y. A regular hexagon of 1″ side lies on this plane with one side inclined at 20° to the H.P. Show its plan and elevation.

11. Draw the plan of a circle $2\frac{1}{2}$″ in diameter when its plane is inclined at 60°.

12. A circle 3′ in diameter lies in a plane inclined at 55° to the paper; draw a plan and an elevation, the ground line making an angle of 60° with the horizontal of that plane.

13. A circle 2·25″ in diameter lies in a plane whose traces both make 45° with X Y. Draw its plan and elevation.

14. Draw the plan of an isosceles triangle whose base is 2·5″ and sides 3″ when that base is inclined at 40° and one of the sides at 48°.

15. Draw the plan and two elevations of an equilateral triangle, A B C, of 3″ side when the sides A B, B C are inclined at 35° and 55° to the paper; one elevation to be on a plane parallel to A C.

16. Two sides of a square are inclined at 30° and 40°. Draw its plan and give an elevation on a line parallel to one diagonal of the plan.

17. Draw the plan of a square of 3″ side when its two diagonals are inclined at 28° and 38°.

18. Draw the plan of a regular hexagon of 1″ side when one of its sides is inclined at 20° and an adjacent side at 25°.

19. A pentagon of 2″ side has one side inclined at 25° and one diagonal meeting that side inclined at 35°. Draw its plan.

20. Draw the plan of an equilateral triangle of 2·5″ side when so held that its corners are ·9″, 1·6″, 2·5″ respectively above the H.P.

21. An isosceles triangle, having its sides 2·5″ and its base 3″, has the two ends of the latter at ·6″, 1·9″, and its vertex 2·8″ above the ground. Draw its plan.

22. The centre of a regular hexagon of 1·5″ side is 2″ above the ground, and the extremities of one side are $\frac{3}{4}$″ and 1″ respectively above the ground. Draw the plan.

23. A square, A B C D, of 2″ side has the corners A, B, C, at heights of $\frac{1}{2}$″, $\frac{3}{4}$″, $1\frac{1}{2}$″ respectively. Draw a plan of the square and an elevation on a vertical plane parallel to the diagonal B D.

24. A rectangle, sides 2″ and 3″, revolves upon one diagonal as a fixed horizontal line, till the plan of a right angle opposite becomes an angle of 120°. What is the inclination of the plane of the figure?

25. A triangle, $a\,b\,c$ ($a\,b = 3''$, $b\,c = a\,c = 2''$), is the plan of an equilateral triangle, A B being horizontal. What is the inclination to the ground of the plane containing it?

26. A rectangle 2·5″ × 1·75″ is the plan of a square. Determine the inclination of the plane of the square.

27. A rhombus ($1\frac{3}{4}''$ side, longest diagonal $2\frac{3}{4}''$) is the plan of a square, one of whose diagonals is horizontal. Find the side of the square and the inclination of its plane.

28. Under what conditions is the plan of a square—(*a*) an equal square, (*b*) a straight line, (*c*) a rectangle, (*d*) a parallelogram, (*e*) a rhombus ?

29. Can any parallelogram drawn at pleasure be the projection of a square ? Give reasons for your answer.

30. Draw a line, $a\,b$, $1{\cdot}6''$ long. From c, the middle point of $a\,b$, draw $c\,d$ $2{\cdot}8''$ long, making the angle $b\,c\,d$ 60°. These two lines represent the plan of a T-square. a and d are on the horizontal plane. Determine the true size of the T-square.

31. Two lines which meet are at right angles to one another and are equally inclined to the H.P. Their plans contain an angle of 110°. Draw an elevation of them on a ground line parallel to the plan of one.

32. Draw a rectangle, $a\,b\,c\,d$. ($a\,b = 2\frac{1}{2}''$, $b\,c = 3\frac{1}{2}''$). From a point, f, in $b\,c$ $\cdot 8''$ from b draw $f\,g$ $2\frac{1}{2}''$ long, making the angle $c\,f\,g$ equal to 20°. On $f\,g$, and within the rectangle, construct an equilateral triangle, $f\,g\,h$. $a\,b\,c\,d$ represents a blackboard and $f\,g\,h$ a triangle drawn upon it. Draw the plan of the board and the triangle when the plane of the former is inclined at 45° and its long edges ($a\,d$, $b\,c$) are horizontal.

33. A triangle, $a\,o\,b$ ($a\,o = 1{\cdot}1''$, $o\,b = 1{\cdot}3$, $a\,b = 1{\cdot}7''$), is the plan of one of the eight isosceles triangles which make up an octagon; o is the plan of the centre of the octagon. Complete the plan of the latter.

34. Draw the plan of a square of $2''$ side when its plane is inclined at 40° and two opposite corners are $1''$ and $1\frac{1}{2}''$ above the ground.

APPENDIX.

Examination Paper on Practical Solid Geometry, set by the Science and Art Department, May 1883.

FIRST STAGE OR ELEMENTARY EXAMINATION.[1]

INSTRUCTIONS.[2]

The constructions may be left in pencil, provided they are distinct and neat, and that the construction lines are shown. They must be strictly geometrical and not the result of calculation or trial. In the absence of those lines which are essential to a correct solution no marks will be awarded, however correct the result may appear.

Lines parallel or perpendicular to others may be drawn mechanically without showing any construction. Lines may be bisected by trial.

Only Eight Questions are to be attempted.

Questions marked (*) have accompanying diagrams.[3]

The examination in this subject lasts for four hours.

Solid Geometry.

1. Represent correctly by their projections on a horizontal and vertical plane:—

   a. A line parallel to both planes of projection.

   b. A line passing through a point on the ground line.

   c. A point equidistant from both planes of projection.

   d. A line equally inclined to both planes of projection. (8.)[4]

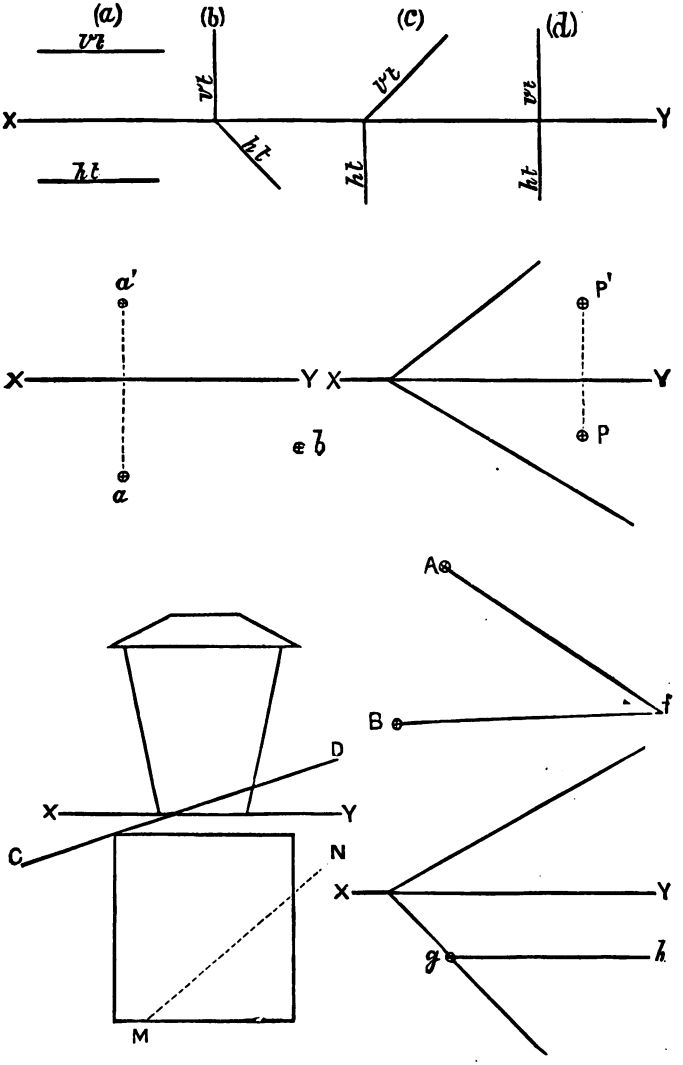
[1] For the Advanced and Honours Papers see Appendix to Part II.

[2] We have only reprinted part of the instructions given. We have also omitted the four problems on Plane Geometry which were set at the same examination.

[3] The diagrams (fig. 114) are drawn to the scale of $\frac{4}{5}$ths.

[4] The relative values of the questions are given by the numbers in *brackets* at the end of each question.

FIG. 114



*2. State precisely what is represented by each of the four given figures, a, b, c, d. (8.)

*3. Determine the traces of any plane not perpendicular to either plane of projection and containing the given point $a\,a'$. (10.)

*4. b is the plan of a point which is $2\frac{3}{4}''$ from the given point $a\,a'$. Determine its elevation b'. (10.)

*5. From the given point $p\,p'$ in the given plane draw two lines in the plane inclined at 29°, and determine the real angle they contain. (14.)

*6. The given lines $A\,f$, $B\,f$ represent the plan of a pair of dividers whose points A, B are in the horizontal plane. The legs are $5''$ long. Determine and write down the height of the joint above the horizontal plane. (12.)

*7. $g\,h$ is the plan of a line lying in the given plane. Determine its elevation and real length. (12.)

8. Draw the projections of a square of $2\frac{3}{4}''$ side; one side to be in the ground line, the opposite side to be equidistant from both planes of projection. (14.)

*9. The figure represents a street lamp. Complete the plan and make an elevation on $C\,D$. (15.)

*10. Make a sectional elevation of the lamp on M N. (15.)

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